Neutrino Physics Prospects of Neutrinoless Double-Beta Decay

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Compelling Evidences for $\nu-$Oscillations

$-\nu_{\rm atm}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ dependences of $\mu-$like events

Dominant $\nu_\mu \rightarrow \nu_\tau$ K2K; MINOS, CNGS

$-\nu_\odot$: Homestake, Kamiokande, SAGE, GALLEX/GNO
Super-Kamiokande, SNO; KamLAND

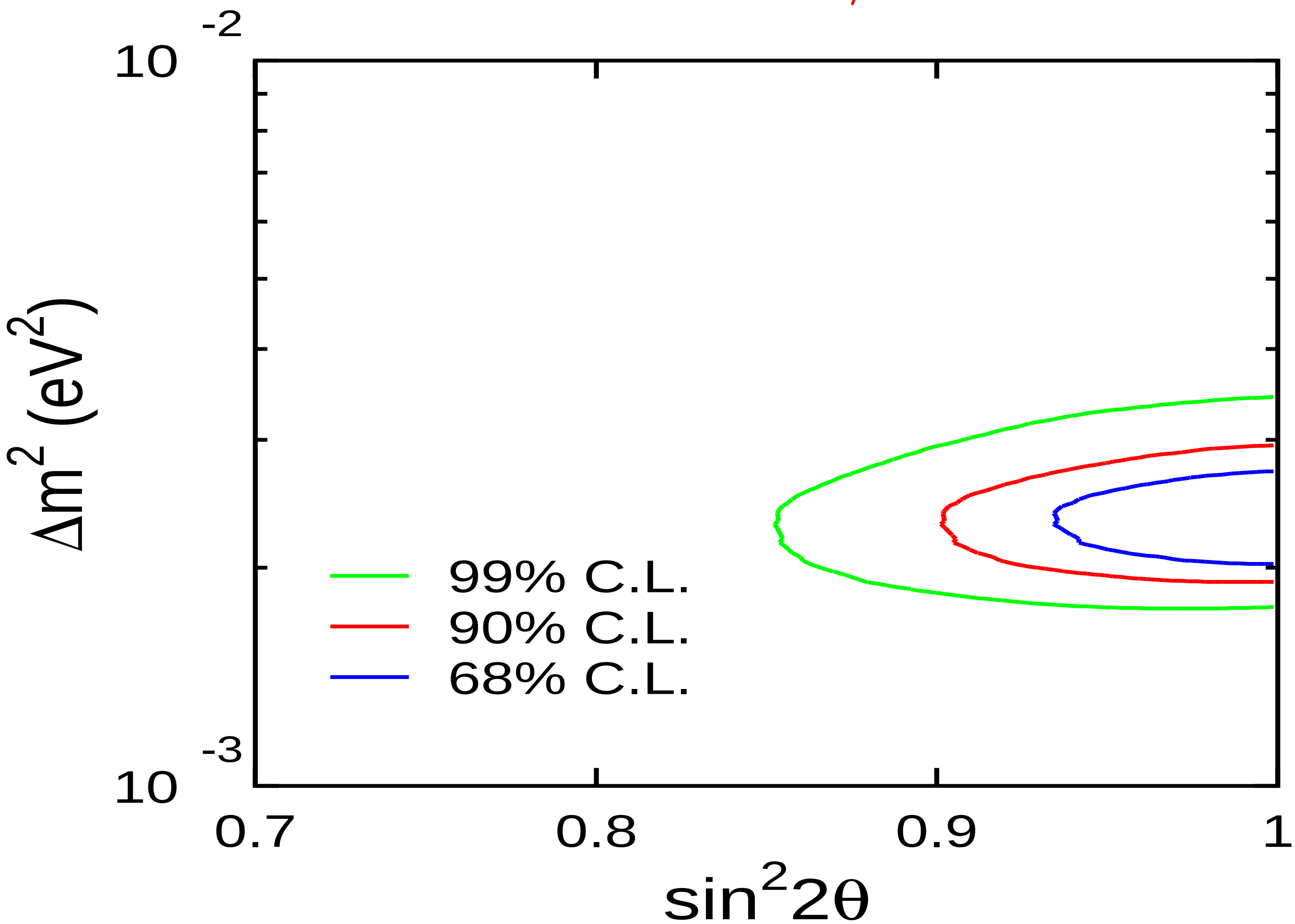
Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ BOREXINO,..., LowNu

– LSND

Dominant $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ MiniBOONE

$$\nu_{l{\rm L}} = \sum_{j=1} U_{lj}\,\nu_{j{\rm L}} \qquad l = e, \mu, \tau. \tag{1}$$

SK: Atmospheric ν Data, L/E



$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} = 2.1 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{\rm atm} \equiv \sin^2 2\theta_{23} = 1.0$;

$\Delta m^2_{31} = (1.3 - 4.2) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \geq 0.85$, 99.73% C.L.

$+$ K2K : $\Delta m^2_{31} = (1.4 - 3.3) \times 10^{-3}$ eV^2, 99.73% C.L.

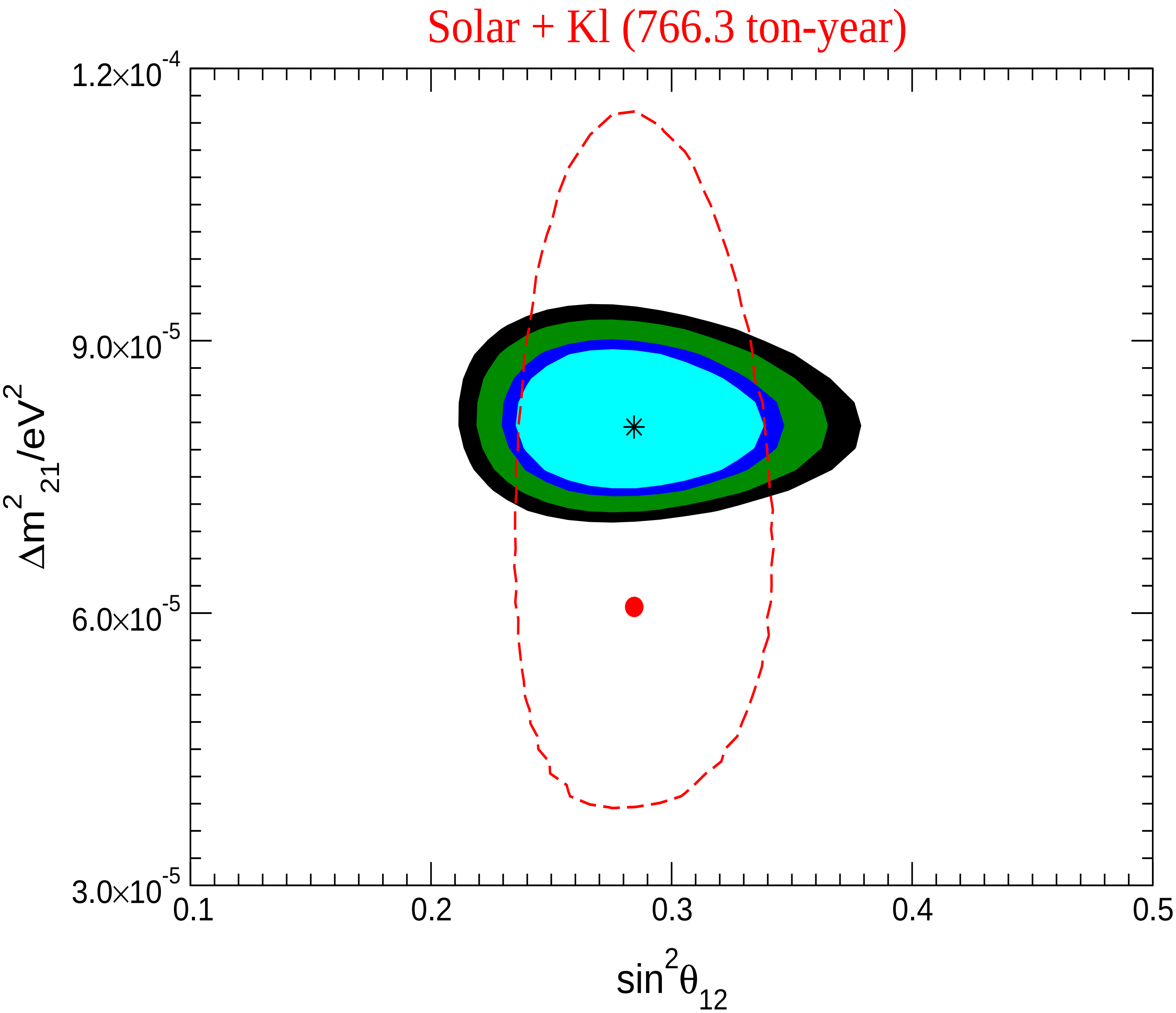
M. Maltoni et al., hep-ph/0405172

- sign of $\Delta m^2_{\rm atm}$ not determined;

  3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (NH);

  $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (IH).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



$\Delta m^2_{\odot} \equiv \Delta m^2_{21} = 8.0 \times 10^{-5}$ eV^2 , $\sin^2\theta_{\odot} \equiv \sin^2\theta_{12} = 0.31$;

$\cos 2\theta_{12} = 0.39$; $\cos 2\theta_{12} > 0.28$,   95% C.L.

- $\sin^2\theta_{12} = 0.50$ excluded at > 6 s.d.

- High-LMA excluded at ~ 3.3 s.d.

A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, hep-ph/0406328;
M. Maltoni et al., hep-ph/0405172

$3-\nu$ Mixing Analysis: $\Delta m^2_\odot \ll |\Delta m^2_{\rm atm}|$

$$P^{3\nu}_\odot \cong \sin^4\theta_{13} + \cos^4\theta_{13}\ P^{2\nu}_\odot,$$

$$P^{2\nu}_\odot = \bar{P}^{2\nu}_\odot + P^{2\nu}_{\odot\ \rm osc},$$

$$\bar{P}^{2\nu}_\odot = \frac{1}{2} + (\frac{1}{2} - P')\cos 2\theta^m_{12}(t_0)\cos 2\theta_{12} \qquad (\theta_{12} \equiv \theta_\odot),$$

W. Haxton, S. Parke, 1986

$$N_e \rightarrow N_e \cos^2\theta_{13},$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E}\sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

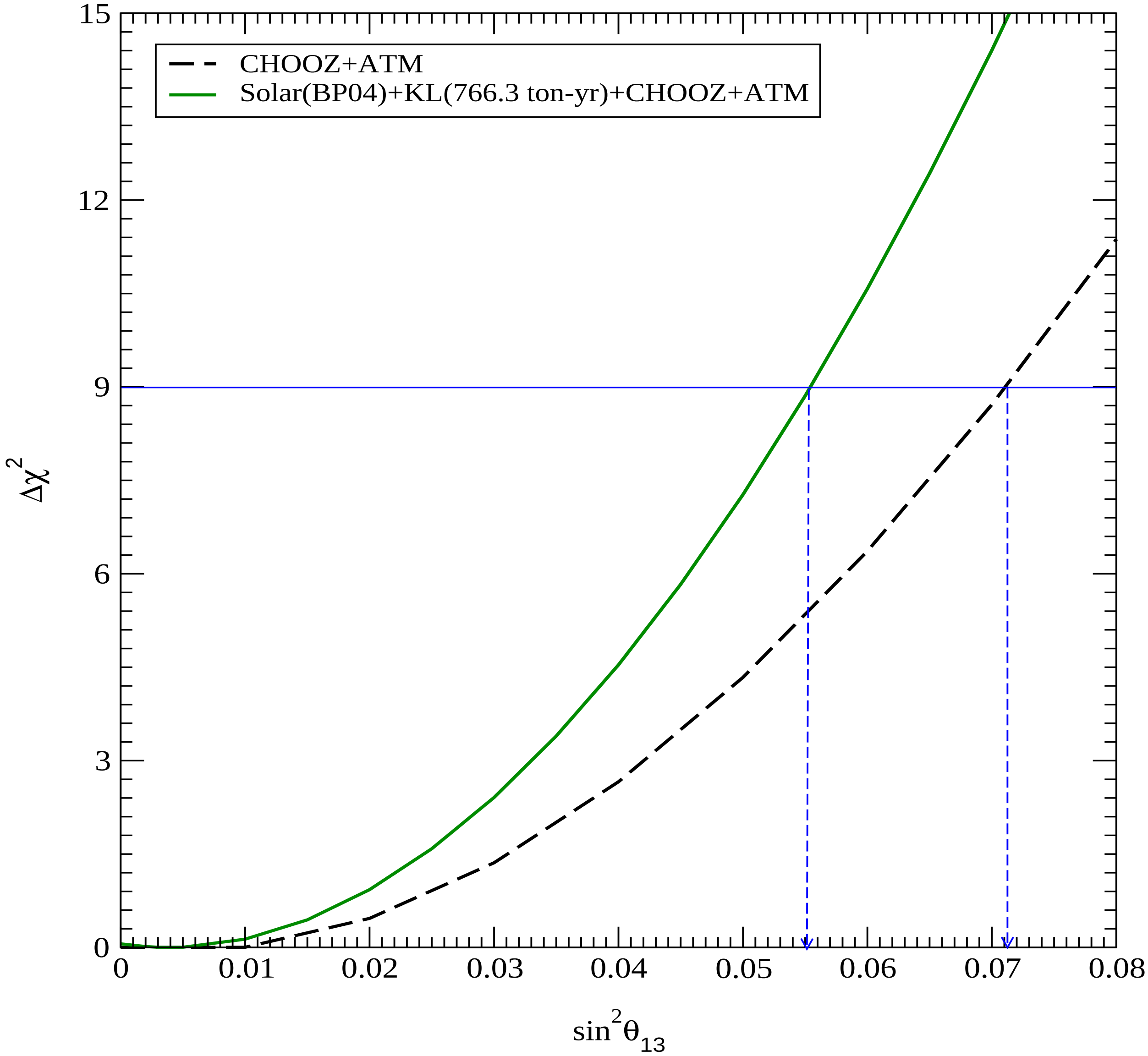
S.T.P., 1988

LMA: $P' \ll 1, \quad < P^{2\nu}_{\odot\ \rm osc} > \cong 0$

J. Rich, S.T.P., 1988

$$P^{3\nu}_{\rm KL} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_\odot}{4E}L)\right]$$

$$P^{3\nu}_{\rm CHOOZ} \cong 1 - \sin^2 2\theta_{13}\sin^2(\frac{\Delta m^2_{\rm atm}}{4E}L)$$



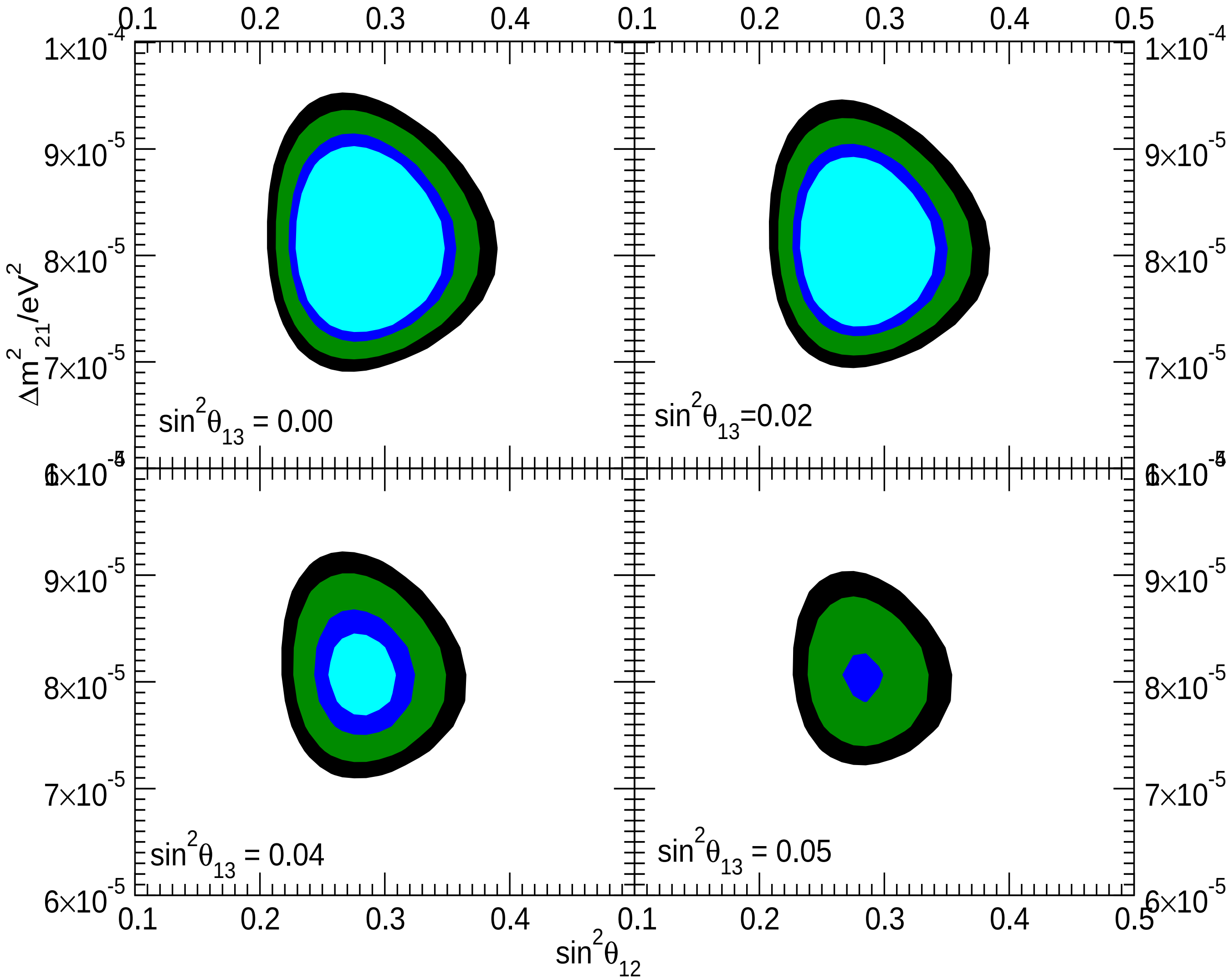
$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} = (1.3 - 4.2) \times 10^{-3}\ \text{eV}^2,\ 3\ s.d.$ (SK : $\nu'04$)

SK: E. Kearns, talk at ν'04, June 2004, Paris

- $\sin^2\theta_{13} < 0.05$ at 99.73% C.L.

A. Bandyopadhyay et al., hep-ph/0406328

Solar + KamLAND(766.3 ton year) + CHOOZ



$\sin^2\theta_{13} = 0$, 95% C.L. :

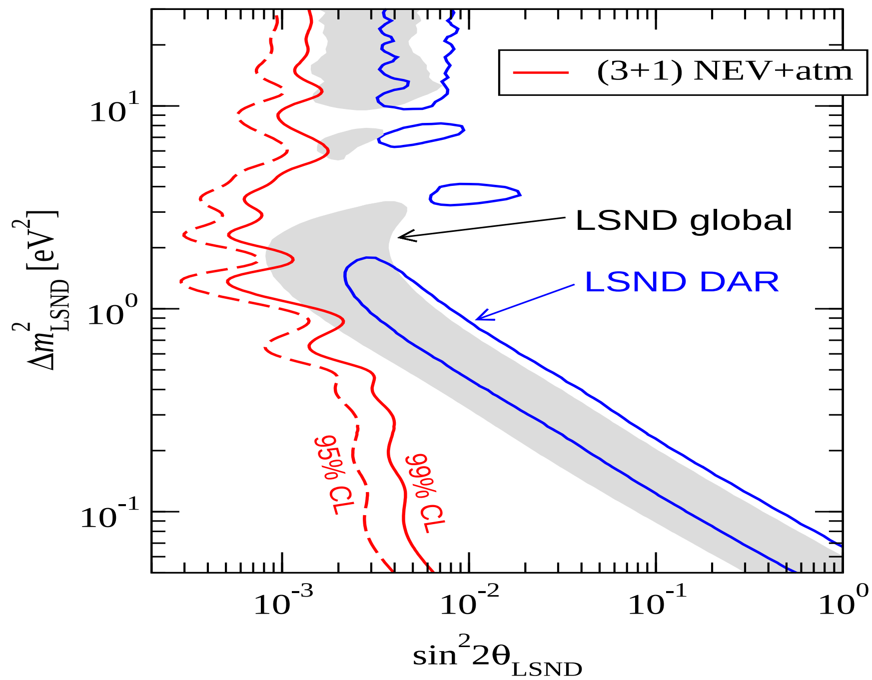
$\Delta m^2_{21} = (7.2 - 8.6) \times 10^{-5}$ eV^2 , $\sin^2\theta_{12} = (0.26 - 0.36)$;

$\cos 2\theta_{12} = 0.40$; $\cos 2\theta_{12} > 0.28$, 95% C.L.;

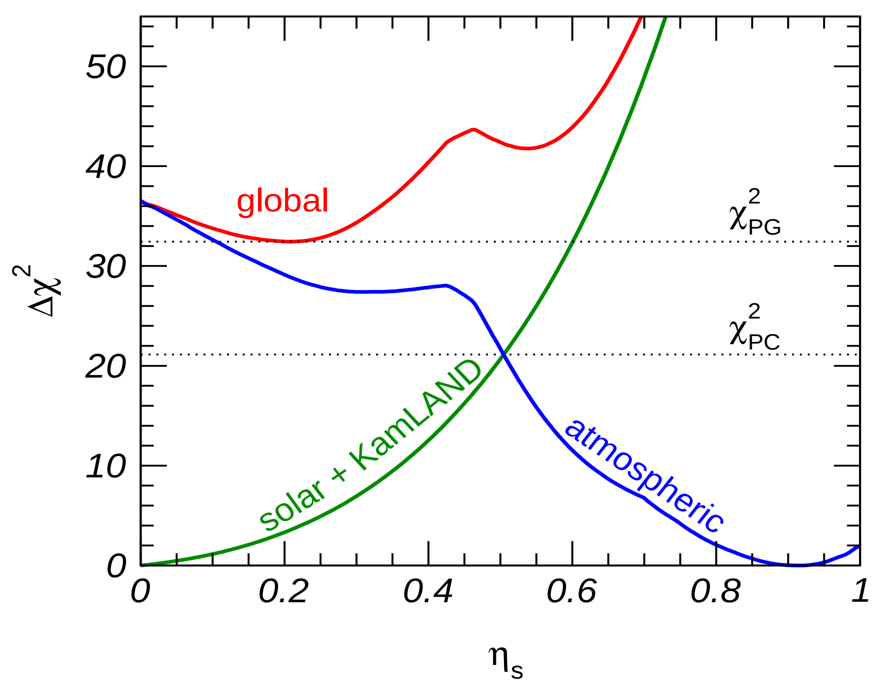
- $\sin^2\theta_{12} = 0.50$ excluded at > 6 s.d.

A. Bandyopadhyay et al, hep-ph/0406328

(3+1)



(2+2)



M. Maltoni et al., hep-ph/0405172

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ . \tag{2}$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{3}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980;
J. Schechter, J.W.F. Valle,1980;
M. Doi, T. Kotani, E. Takasugi,1981

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \to e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\mathrm{el}} = 0,\ L_l = 0,\ L = 0, \ldots$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0\ , \quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0\ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta}\ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi\ C^{-1}_{\alpha\kappa} S^F_{\kappa\beta}(x-y)$$

$$U_{CP}\ \chi(x)\ U^{-1}_{CP} = \eta_{CP}\ \gamma_0\ \chi(x'), \quad \eta_{CP} = \pm i\ .$$

Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix} \quad (4)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \quad (5)$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- If $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$, $\Delta m^2_{\text{atm}} = \Delta m^2_{31}$,

then $\theta_{12} = \theta_{\odot}$, $\theta_{23} = \theta_{\text{atm}}$, $\theta_{13} = \theta$.

The angle θ_{13} is limited by the data from the CHOOZ and Palo Verde experiments.

- α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987
  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$?

Neutrino Mixing Parameters

$\theta_{12},\ \theta_{23},\ \theta_{13}$

ν_j	Dirac	Majorana
	δ	$\delta,\ \alpha_{21},\ \alpha_{31}$

$m_1, \quad m_2, \quad m_3$

$m_1,\ m_2,\ m_3$ - in terms of $\Delta m^2_{\odot}$, $\Delta m^2_{\rm atm}$ and $\min(m_j)$

Conventions

A. $m_1 < m_2 < m_3$ (NH) or $m_3 < m_1 < m_2$ (IH)

- $\Delta m^2_{\odot} = \Delta m^2_{21} > 0$
- $\Delta m^2_{\rm atm} = \Delta m^2_{31} > 0$ (NH), $\Delta m^2_{\rm atm} = \Delta m^2_{32} < 0$ (IH)
- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$

B. $m_1 < m_2 < m_3$

- $\Delta m^2_{\rm atm} = \Delta m^2_{31} > 0$

Two possibilities:

$\Delta m^2_{\odot} = \Delta m^2_{21} > 0$, NH

$\Delta m^2_{\odot} = \Delta m^2_{32} > 0$, IH

"discrete" parameter

Future Progress

- High precision determination of $\Delta m^2_\odot$, $\theta_\odot$, Δm^2_{atm}, θ_{atm}.

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- Determination of the type of the $\nu-$ mass spectrum

$$m_1 \ll m_2 \ll m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.20 \text{ eV}.$$

- Determining or obtaining significant constraints on the absolute scale of neutrino masses, or on $min(m_j)$.

- Determination of the nature - Dirac or Majorana, of ν_j .

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation, i.e., finding The Theory of $\nu-$mixing.

$(\beta\beta)_{0\nu}$−Decay Experiments:

- Majorana nature of ν_j
- Type of ν−mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

^{3}H β-decay, cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

ν_j− Dirac or Majorana particles,

fundamental problem

ν_j−Dirac: **conserved lepton charge exists,**

$$L = L_e + L_\mu + L_\tau, \qquad \nu_j \neq \bar{\nu}_j$$

ν_j−Majorana: **no lepton charge is exactly conserved.**

$\nu_j \equiv \bar{\nu}_j$ − **fermionic analogs of π^0**

The observed patterns of ν−mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_\odot$ can be related to Majorana ν_j and an approximate symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j− Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of ν− oscillations.

If $\nu_j -$ Majorana particles,
U_{PMNS} **contains (3-ν mixing)**

δ-Dirac, α_{21}, α_{31} - Majorana **physical CPV phases**

ν-oscillations $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$,

- **are not sensitive to the nature of ν_j,**

S.M. Bilenky, J. Hosek, S.T.P.,1980;
P. Langacker et al., 1987

- **provide information on $\Delta m^2_{jk} = m^2_j - m^2_k$, but not on the absolute values of ν_j masses.**

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:

$$K^+ \rightarrow \pi^- + \mu^+ + \mu^+$$
$$\mu^- + \text{ (A, Z)} \rightarrow \mu^+ + \text{ (A, Z} - 2)$$

The process most sensitive to the possible Majorana nature of ν_j - $(\beta\beta)_{0\nu}$-decay

$$\text{(A, Z)} \rightarrow \text{(A, Z} + 2) + e^- + e^-$$

of certain even-even nuclei, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe.

2n from (A,Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into 2p of (A,Z+2) and two free e^-.

$A(\beta\beta)_{0\nu} \sim <m>$ M(A,Z), M(A,Z) - NME,

$$|<m>| = \left|m_1|U_{e1}|^2 + m_2|U_{e2}|^2\ e^{i\alpha_{21}} + m_3|U_{e3}|^2\ e^{i\alpha_{31}}\right|,$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi$, $\alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2, and of ν_1 and ν_3 .

L. Wolfenstein, 1981;
S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;
B. Kayser, 1984.

Best sensitivity: Heidelberg-Moscow ^{76}Ge experiment.

Claim for a positive signal at $> 3\sigma$:
H. Klapdor-Kleingrothaus et al., PL B586 (2004),
$|<m>| = (0.1 - 0.9)$ eV (99.73% C.L.).

IGEX ^{76}Ge: $|<m>| < (0.33 - 1.35)$ eV (90% C.L.).

Taking data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|<m>| <(0.7$–$1.2)$ eV, $|<m>| <(0.2$–$1.1)$ eV (90% C.L.).

Large number of projects: $|<m>| \sim (0.01 - 0.05)$ eV
CUORE - ^{130}Te,
GERDA - ^{76}Ge,
EXO - ^{136}Xe,
MAJORANA - ^{76}Ge,
MOON - ^{100}Mo,
CANDLES - ^{48}Ca,
XMASS - ^{136}Xe.

$|<m>|$: m_j, $|U_{ej}|^2$, $\alpha_{21,31}$

$m_{1,2,3}$ - in terms of $\min(m_j)$, Δm^2_{atm}, $\Delta m^2_{\odot}$

$|U_{ej}|^2$ - in terms of $\sin^2\theta_{\odot}$, $\cos^2\theta_{\odot}$, $\sin^2\theta$ (CHOOZ)

In convention (B) in which always $m_1 < m_2 < m_3$,

$$\Delta m^2_{atm} \equiv \Delta m^2_{31}, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{atm}},$$

while either

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21} \quad \text{(normal mass hierarchy)} \quad \text{or}$$

$$\Delta m^2_{\odot} \equiv \Delta m^2_{32} \quad \text{(inverted mass hierarchy)}.$$

Normal mass hierarchy: $m_2 = \sqrt{m_1^2 + \Delta m^2_{\odot}}$,

$$|U_{e1}|^2 = \cos^2\theta_{\odot}(1 - |U_{e3}|^2), \quad |U_{e2}|^2 = \sin^2\theta_{\odot}(1 - |U_{e3}|^2),$$

$$|U_{e3}|^2 \equiv \sin^2\theta \quad \text{(CHOOZ)}$$

Inverted mass hierarchy: $m_2 = \sqrt{m_1^2 + \Delta m^2_{atm} - \Delta m^2_{\odot}}$,

$$|U_{e2}|^2 = \cos^2\theta_{\odot}(1 - |U_{e1}|^2), \quad |U_{e3}|^2 = \sin^2\theta_{\odot}(1 - |U_{e1}|^2),$$

$$|U_{e1}|^2 \equiv \sin^2\theta \quad \text{(CHOOZ)}.$$

The neutrino mass spectrum –

normal hierarchical (NH) if $m_1 << m_2 << m_3$,

inverted hierarchical (IH) if $m_1 << m_2 \cong m_3$.

The *quasi-degenerate (QD)* mass spectrum is realized if $m_1 \cong m_2 \cong m_3$, $m^2_{1,2,3} >> \Delta m^2_{atm}$.

Given Δm^2_{atm}, $\Delta m^2_{\odot}$, $\theta_{\odot}$, θ,

$$|<m>| = |<m>| \; (\mathrm{m_{min}}, \alpha_{21}, \alpha_{31}; \mathrm{S}), \; S = NH, IH.$$

Oscillation Parameters

$$\Delta m^2_\odot = 8.0 \times 10^{-5}\ \text{eV}^2\ ,\quad 3\sigma(\Delta m^2_\odot) = 12\%\ ,$$

$$\sin^2\theta_\odot = 0.31\ ,\quad 3\sigma(\sin^2\theta_\odot) = 24\%\ ,$$

$$|\Delta m^2_{\text{atm}}| = 2.2 \times 10^{-3}\ \text{eV}\ ,\quad 3\sigma(|\Delta m^2_{\text{atm}}|) = 50\%.$$

Future:

SNO III: $3\sigma(\sin^2\theta_\odot) = 21\%$;

3 kTy KamLAND: $3\sigma(\Delta m^2_\odot) = 7\%$, $3\sigma(\sin^2\theta_\odot) = 18\%$;

A. Bandyopadhyay et al., hep-ph/0410283

SK-Gd (0.1% Gd: 43×(KL $\bar{\nu}_e$ rate)), 3y: $3\sigma(\Delta m^2_\odot) \cong 4\%$

S. Choubey, S.T.P., hep-ph/0404103;
J. Beacom and M. Vagins, hep-ph/0309300

KL type reactor $\bar{\nu}_e$ detector, $L \sim 60$ km, ~ 60 GW kTy:

$3\sigma(\sin^2\theta_\odot) \cong 12\%$

A. Bandyopadhyay et al., hep-ph/0410283 and hep-ph/0302243;
H. Minakata et al., hep-ph/0407326

T2K (SK): $3\sigma(|\Delta m^2_{\text{atm}}|) \cong 12\%$

P. Huber et al., hep-ph/0403068

$\text{sgn}(\Delta m^2_{\text{atm}})$: atmospheric ν experiments, studying the sub-dominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations; LBL $\nu-$oscillation experiments (T2K, NOνA); $\nu-$factory.

$\sin^2\theta_{13}$: reactor $\bar{\nu}_e$ experiments, $L \sim (1-2)$ km: Double CHOOZ, Braidwood, Daya-Bay, KASKA - factor (5 - 10).

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decayexperiments

$$m_{\nu_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN :} \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < 0.70 \text{ eV} \quad (95\% \text{ C.L.}) \quad (X \sim 3)$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Normal Hierarchical Spectrum, $m_1 \ll m_2 \ll m_3$:

$$m_2 \cong \sqrt{\Delta m^2_{\odot}} \cong (8.4 - 9.4) \times 10^{-3}\ \text{eV} \quad (3\sigma),$$

$$m_3 \cong \sqrt{\Delta m^2_{\text{atm}}} \cong (3.7 - 5.7) \times 10^{-2}\ \text{eV} \quad (3\sigma).$$

Inverted Hierarchical Spectrum, $m_3 \ll m_1 \cong m_2$:

$$m_{1,2} \cong \sqrt{|\Delta m^2_{\text{atm}}|} \cong (3.7 - 5.7) \times 10^{-2}\ \text{eV}\ .$$

Quasi-Degenerate Spectrum, $m_1 \cong m_2 \cong m_3 \equiv m$:

$$m^2_{1,2,3} >> |\Delta m^2_{\text{atm}}|\ .$$

Using $|\Delta m^2_{\text{atm}}|$ and $\Delta m^2_{\odot}$ inferred from the data one has

Normal (Inverted) Hierarchical Spectrum for

$$m_1 << 0.02\ \text{eV} \quad (m_3 < 0.02\ \text{eV})\ ;$$

Spectrum with Partial Hierarchy for

$$0.02\ \text{eV} \lesssim m_{1(3)} \lesssim 0.20\ \text{eV}\ ;$$

Quasi-Degenerate Spectrum for

$$m_{1(3)} \gtrsim 0.20\ \text{eV}\ .$$

Hierarchical Neutrino Mass Spectrum

$$m_1 \ll m_2 \ll m_3.$$

The pattern corresponds to

$$m_1 \ll \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{32}} \ .$$

It is possible to identify

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\text{atm}} \equiv \Delta m^2_{32},$$

$$|U_{\text{e}1}|^2 \ = \cos^2\theta_{\odot}(1 - \ |U_{\text{e}3}|^2 \ ), \quad |U_{\text{e}2}|^2 \ = \sin^2\theta_{\odot}(1 - \ |U_{\text{e}3}|^2 \ ),$$

$$|U_{\text{e}3}|^2 \ \equiv \sin^2\theta_{13} < 0.05 \qquad (\text{CHOOZ } + \nu_{\text{A}} + \nu_{\odot} + \text{ KL}).$$

This implies:

$$m_2 \simeq \sqrt{\Delta m^2_{\odot}}, \qquad m_3 \simeq \sqrt{\Delta m^2_{\text{atm}}} \ .$$

One has

$$|<m>| \ = \Big|(m_1 \cos^2\theta_{\odot} + \sqrt{\Delta m^2_{\odot}} \sin^2\theta_{\odot})(1 - |U_{\text{e}3}|^2)e^{i\alpha_{21}}$$

$$+ \sqrt{\Delta m^2_{\text{atm}}}|U_{\text{e}3}|^2 e^{i\alpha_{31}}\Big|$$

$$\simeq \left|\sqrt{\Delta m^2_{\odot}}(1 - |U_{\text{e}3}|^2) \sin^2\theta_{\odot} + \sqrt{\Delta m^2_{\text{atm}}}|U_{\text{e}3}|^2 e^{i(\alpha_{31}-\alpha_{21})}\right|$$

Even if $m_1 = 0$, $|<m>|$ depends on Majorana CPV phase $\alpha_{32} = \alpha_{31} - \alpha_{21}$. Reflects the fact that in contrast to the case of massive Dirac ν's (or quarks), CPV can take place in the mixing of only 2 massive Majorana ν's.

Inverted Hierarchical $\nu-$Mass Spectrum

$$m_3 \ll m_1 \simeq m_2.$$

Can also be defined by

$$m_3 \ll \;(<)\; \sqrt{\Delta m^2_{21}} \ll \sqrt{\Delta m^2_{23}}.$$

We can identify

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\rm atm} \equiv \Delta m^2_{32} \simeq \Delta m^2_{31},$$

$$|U_{\rm e3}|^2 = \sin^2\theta_{13} < 0.05 \qquad ({\rm CHOOZ}\; + \nu_{\rm A} + \nu_{\odot} + \;{\rm KL}),$$

$$|U_{\rm e1}|^2 = \cos^2\theta_{\odot}(1 - |U_{\rm e3}|^2), \quad |U_{\rm e2}|^2 = \sin^2\theta_{\odot}(1 - |U_{\rm e3}|^2),$$

$$m_1 \simeq m_2 \simeq \sqrt{|\Delta m^2_{\rm atm}|}.$$

Neglecting $m_3|U^2_{\rm e3}|$ in $|<m>|$,

$$|<m>| \;\cong \sqrt{|\Delta m^2_{\rm atm}|}(1 - |U_{\rm e3}|^2)\sqrt{1 - \sin^2 2\theta_{\odot}\sin^2\left(\frac{\alpha_{21}}{2}\right)},$$

$$\sqrt{|\Delta m^2_{\rm atm}|}(1- |U_{\rm e3}|^2)|\cos 2\theta_{\odot}| \le |<m>| \le \sqrt{|\Delta m^2_{\rm atm}|}(1- |U_{\rm e3}|^2).$$

The upper and lower limits - CP-conserving:

$\alpha_{21} = 0, \qquad \alpha_{21} = \pm\pi.$

$$\sin^2\frac{\alpha_{21}}{2} = \left(1 - \frac{|<m>|^2}{|\Delta m^2_{\rm atm}|(1 - |U_{\rm e3}|^2)^2}\right)\frac{1}{\sin^2 2\theta_{\odot}}.$$

Three Quasi-Degenerate Neutrinos

$$m_1 \simeq m_2 \simeq m_3 \equiv m, \quad m^2 \gg |\Delta m^2_{\text{atm}}|.$$

As for the NH neutrino mass spectrum we have:

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}, \quad \Delta m^2_{\text{atm}} \equiv \Delta m^2_{31},$$

$$|U_{\text{e1}}|^2 = \cos^2\theta_{\odot}(1 - |U_{\text{e3}}|^2), \quad |U_{\text{e2}}|^2 = \sin^2\theta_{\odot}(1 - |U_{\text{e3}}|^2),$$

$$|U_{\text{e3}}|^2 = \sin^2\theta_{13} < 0.05 \quad (\text{CHOOZ} + \nu_{\text{A}} + \nu_{\odot} + \text{KL}).$$

The mass scale m effectively coincides with the $\bar{\nu}_e$ mass $m_{\bar{\nu}_e}$ measured in the current ^{3}H β-decay experiments:

$$m \cong m_{\bar{\nu}_e}.$$

Thus, $m < 2.2$ eV. Cosmology: $m \lesssim 0.6$ eV.

The QD spectrum - realized for m, which can be measured in the ^{3}H $\beta-$decay experiment KATRIN, $m_{\bar{\nu}_e} \gtrsim (0.2-0.3)$ eV.

The effective Majorana mass $|<m>|$ is given by

$$|<m>| \cong m \left|\cos^2\theta_{\odot}(1 - |U_{\text{e3}}|^2) + \sin^2\theta_{\odot}(1 - |U_{\text{e3}}|^2)e^{i\alpha_{21}} + |U_{\text{e3}}|^2 e^{i\alpha_{31}}\right|$$

$$\cong m \left|\cos^2\theta_{\odot} + \sin^2\theta_{\odot} e^{i\alpha_{21}}\right|;$$

$m\,|\cos 2\theta_{\odot}| \lesssim |<m>| \lesssim m$; limits: $\alpha_{21} = 0$; $\pm\pi$ - CPC

$$\sin^2\frac{\alpha_{21}}{2} \cong \left(1 - \frac{|<m>|^2}{m(1 - |U_{\text{e3}}|^2)^2}\right)\frac{1}{\sin^2 2\theta_{\odot}}.$$

Solar neutrino and KamLAND data:

$\cos 2\theta_{\odot} = 0.0$ excluded at > 6 s.d.

Best fit value: $\cos 2\theta_{\odot} \simeq 0.40$

$\cos 2\theta_{\odot} \gtrsim 0.28$, 95% C.L.

Normal hierarchical spectrum:

$$(|<m>|\ )_{\text{max}} \lesssim 0.005 \text{ eV}$$

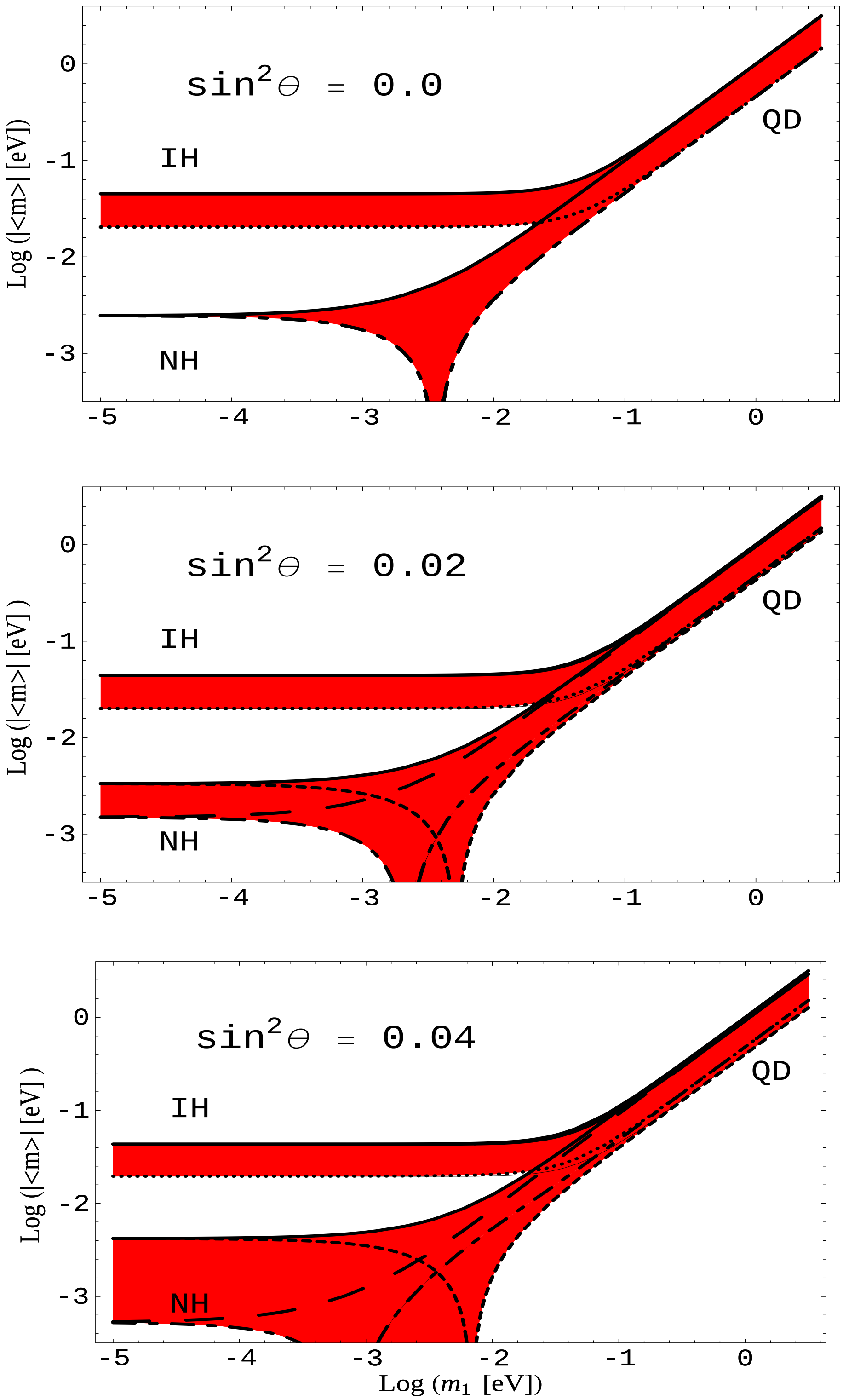
Inverted hierarchical spectrum:

$$(|<m>|\ )_{\text{min}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos 2\theta_{\odot} \cos^2\theta_{13} \gtrsim 0.01 \text{ eV}$$

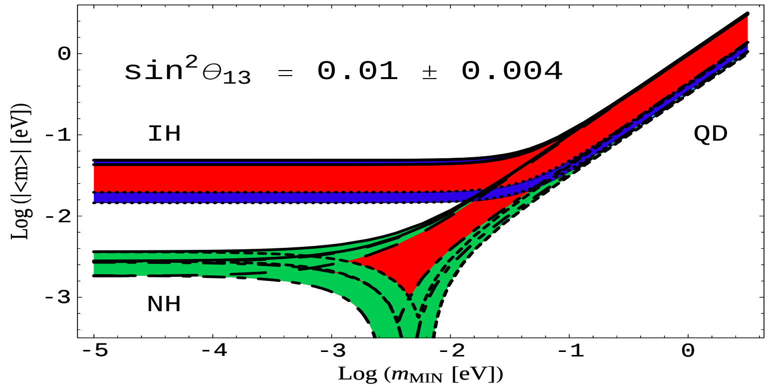
$$(|<m>|\ )_{\text{max}} \simeq \sqrt{|\Delta m^2_{\text{atm}}|} \cos^2\theta_{13} \lesssim 0.055 \text{ eV}$$

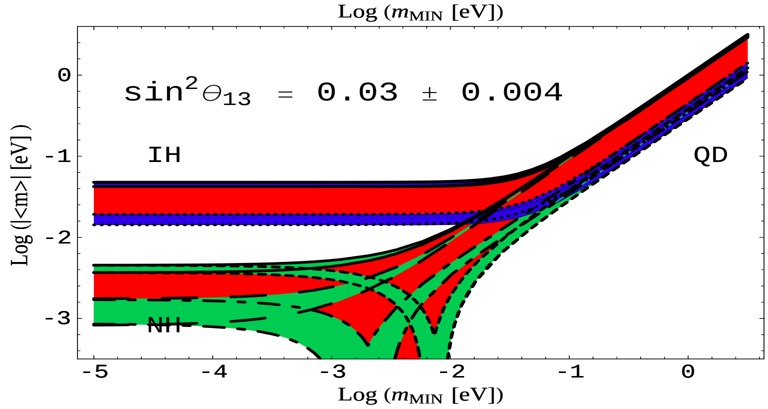
Quasi-degenerate spectrum:

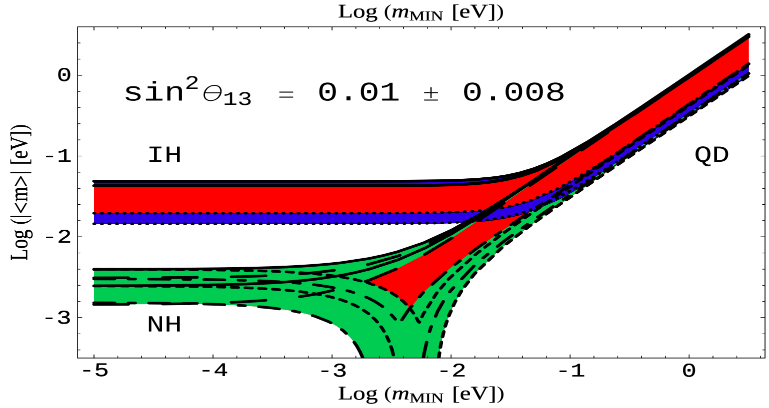
$$(|<m>|\ )_{\text{min}} \simeq m\ (\cos 2\theta_{\odot} \cos^2\theta_{13} - \sin^2\theta_{13}) \gtrsim 0.06 \text{ eV}$$

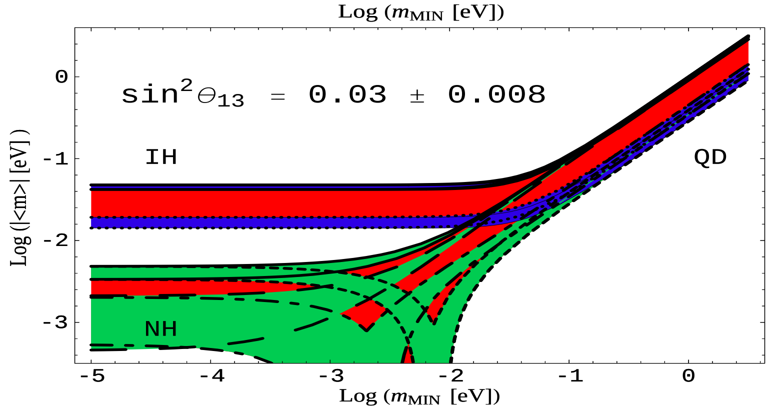


S. Pascoli, S.T.P., 2004









S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Nuclear Matrix Element Uncertainty

$$|<m>| = \zeta \, ((|<m>|_{\rm exp})_{\rm MIN} \pm \Delta) \; , \quad \zeta \geq 1,$$

$(|<m>|_{\rm exp})_{\rm MIN}$– obtained with the **maximal physically allowed value of NME.**

A measurement of the $(\beta\beta)_{0\nu}$-decay half-life time

$$(|<m>|_{\rm exp})_{\rm MIN} - \Delta \leq |<m>| \leq \zeta((|<m>|_{\rm exp})_{\rm MIN} + \Delta) \; .$$

The currently estimated range of ζ^2:

^{48}Ca, $\zeta^2 \simeq 3.5$

^{76}Ge, $\zeta^2 \simeq 10$

^{82}Se, $\zeta^2 \simeq 10$

^{130}Te, $\zeta^2 \simeq 38.7$

S. Elliot, P. Vogel, 2002

NH vs IH (QD):

$$\zeta \; |<m>|^{\rm NH}_{\rm max} < |<m>|^{\rm IH(QD)}_{\rm min} \; , \; \zeta \geq 1 \; .$$

IH vs QD:

$$\zeta \; |<m>|^{\rm IH}_{\rm max} < |<m>|^{\rm QD}_{\rm min} \; , \; \zeta \geq 1 \; .$$

These conditions imply upper limits on $\tan^2\theta_\odot$, which are functions of the neutrino oscillation parameters and of ζ.

S. Pascoli, S.T.P., W. Rodejohann, 2003

Method of Analysis

$$\Gamma_{\text{th}} = G\,|\mathcal{M}|^2\,\left(|{<}\,m\,{>}|\;(\mathbf{x})\right)^2\,,\;\;\mathbf{x} = (\mathbf{x}_{\text{osc}}, \mathbf{x}^{0\nu}_{\beta\beta})$$

$$\mathbf{x}_{\text{osc}} = (\theta_{12}, \theta_{13}, |\Delta m^2_{31}|, \Delta m^2_{21})\,,$$

$$\mathbf{x}^{0\nu}_{\beta\beta} = (m_0, \text{sgn}(\Delta m^2_{31}), \alpha_{21}, \alpha_{31})\,.$$

$$|{<}\,m\,{>}|^{\;\text{obs}} \equiv \sqrt{\frac{\Gamma_{\text{obs}}}{G}}\,\frac{1}{|\mathcal{M}_0|}\,, \quad \sigma_{\beta\beta} = \frac{1}{2}\,\frac{1}{\sqrt{\Gamma_{\text{obs}} G}}\,\frac{1}{|\mathcal{M}_0|}\,\sigma(\Gamma_{\text{obs}})\,,$$

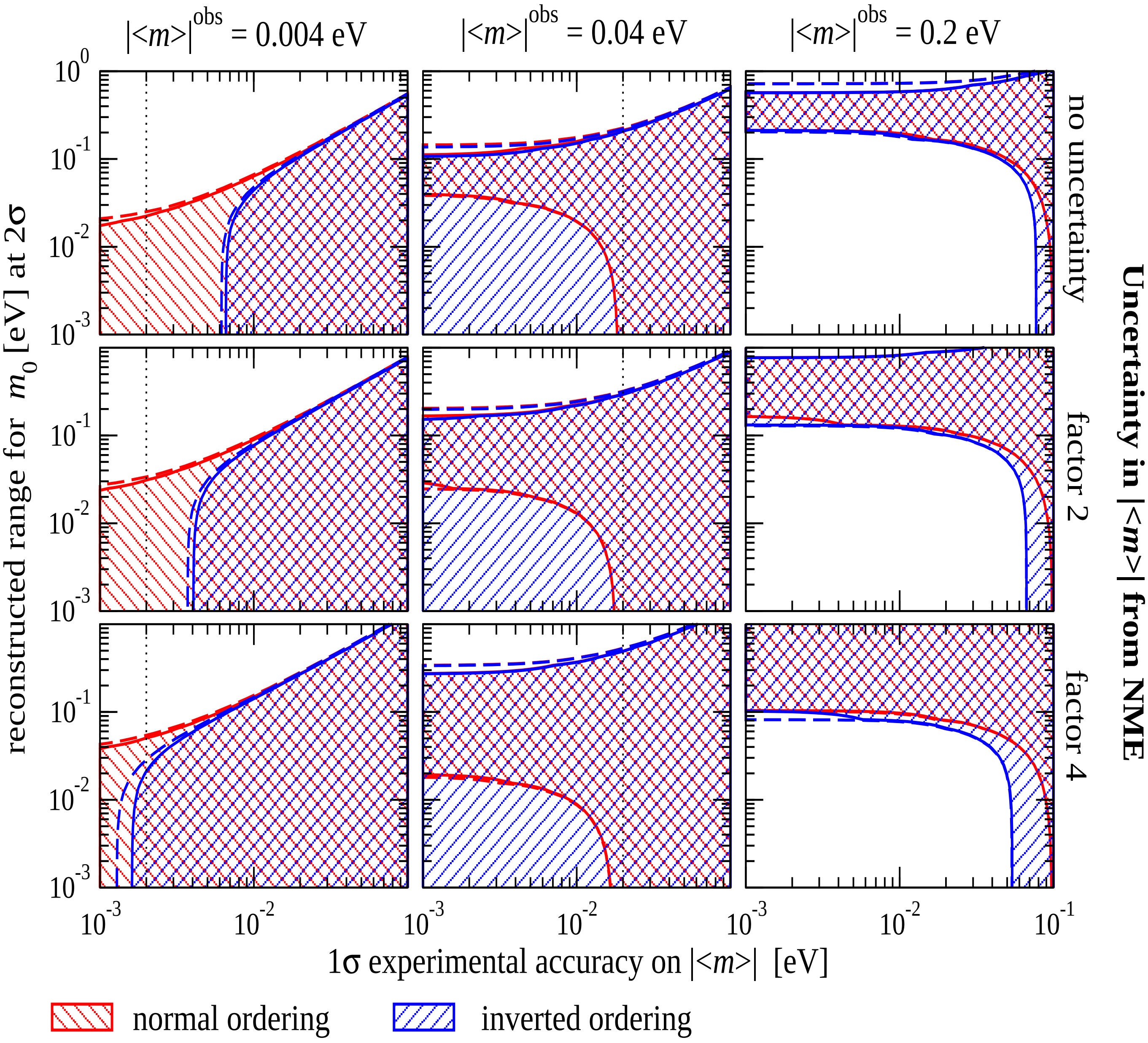
$|\mathcal{M}_0|$ is some nominal value of the NME.

$$\chi^2(\mathbf{x}^{0\nu}_{\beta\beta}, \mathbf{F}) = \min_{\xi \in [1/\sqrt{F}, \sqrt{F}]} \frac{\left[\xi\;|{<}\,m\,{>}|\;(\mathbf{x}) - |{<}\,m\,{>}|^{\;\text{obs}}\right]^2}{\sigma^2_{\beta\beta} + \xi^2 \sigma^2_{\text{th}}}\,.$$

$$\xi \equiv \frac{|\mathcal{M}|}{|\mathcal{M}_0|}\,, \quad \xi = [1/\sqrt{F}, \sqrt{F}], \quad F \geq 1,$$

$|\mathcal{M}|$ is the *true* value of the NME.

Absolute Neutrino Mass Scale

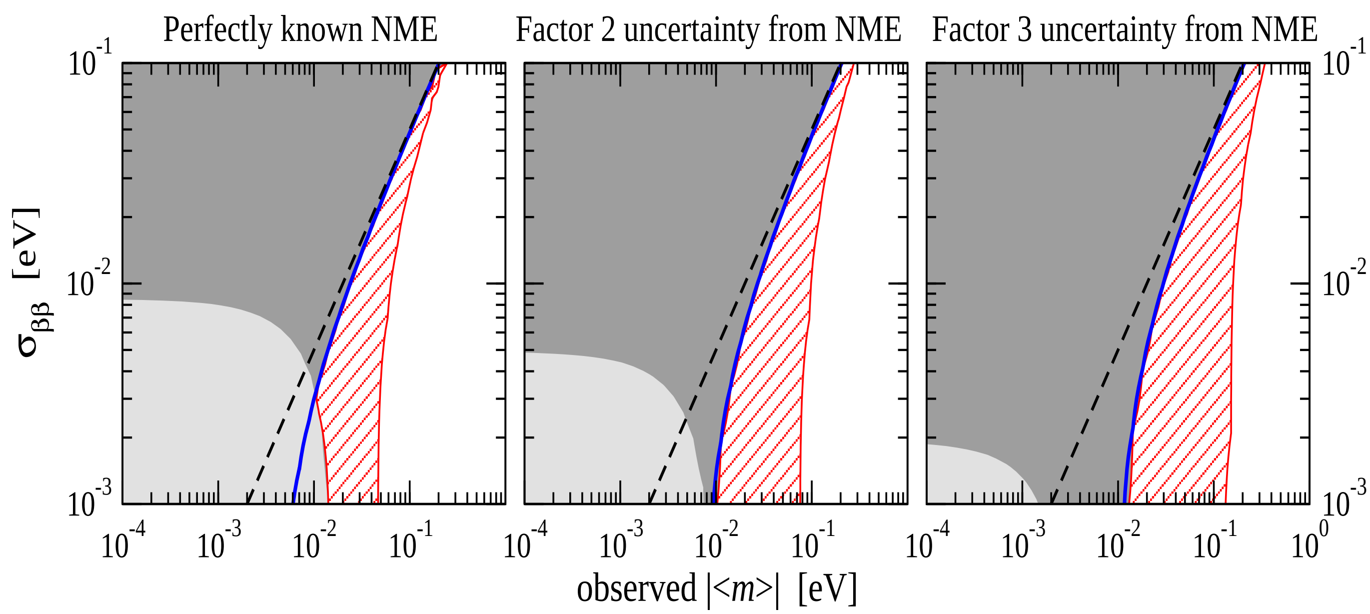


dashed: $\sigma(\sin^2\theta_{13}) = 0.016$, $\sigma(\sin^2\theta_{12}) = 7.5\%$, $\sigma(\Delta m^2_{21}) = 4\%$, $\sigma(\Delta m^2_{31}) = 13\%$ $\sin^2\theta_{13} = 0$

solid: $\sigma(\sin^2\theta_{13}) = 0.002$, $\sigma(\sin^2\theta_{12}) = 3.0\%$, $\sigma(\Delta m^2_{21}) = 2\%$, $\sigma(\Delta m^2_{31}) = 5\%$ $\sin^2\theta_{12} = 0.31$

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Distinguishing Between Different Spectra

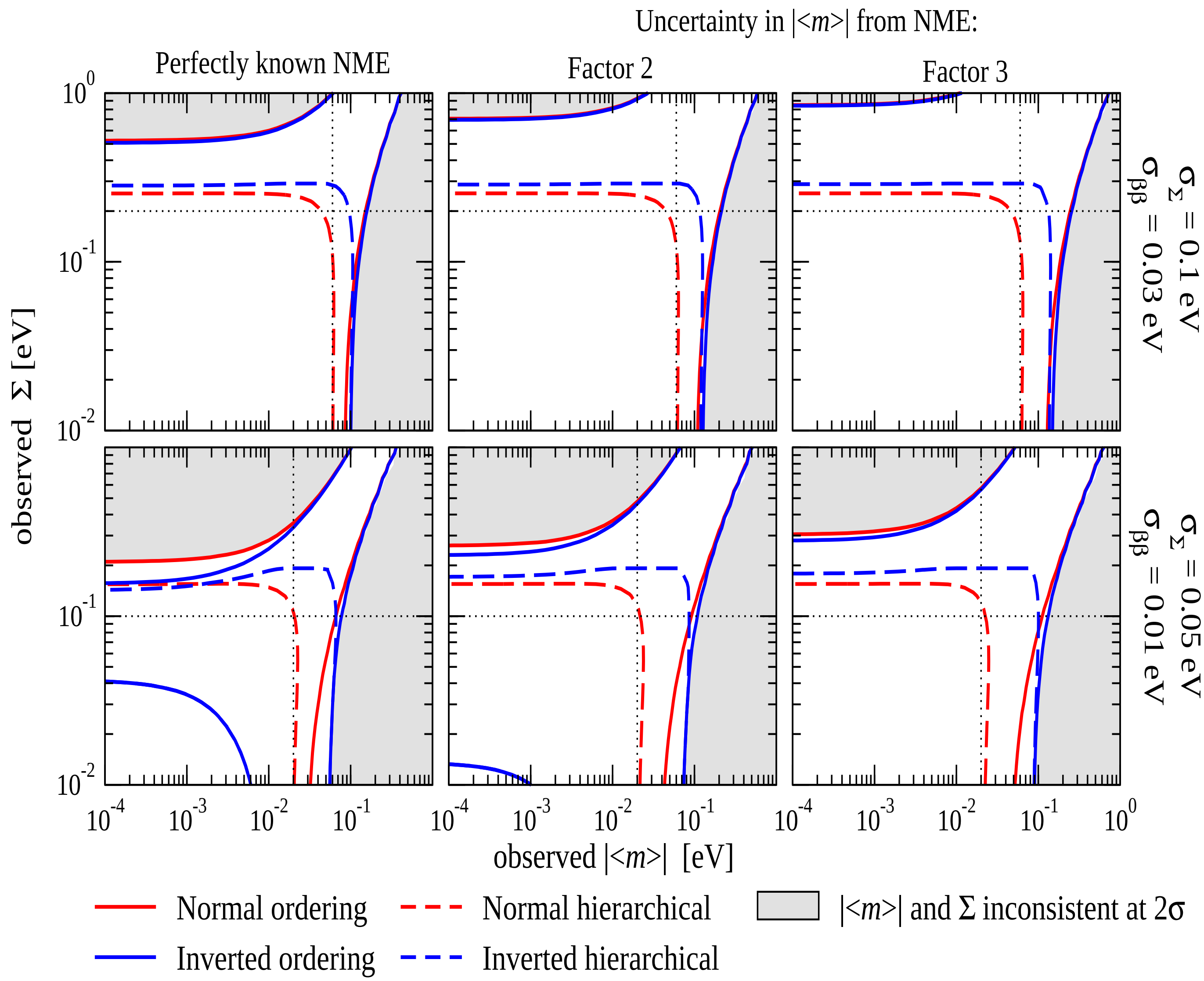


- No information on the mass ordering
- Inverted ordering excluded at 2σ
- To the right a signal is observed at 2σ
- Either IH or QD spectrum
- QD with no information on ordering
- To the right the NH spectrum is excluded

$\sin^2\theta_{13} = 0.03 \pm 0.006$, $\sin^2\theta_{12} = 0.31 \pm 3\%$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $|\Delta m^2_{31}| = 2.2\text{x}10^{-3} \pm 3\%$

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Distinguishing Between Different Spectra



$\sin^2\theta_{13} = 0 \pm 0.002$, $\sin^2\theta_{12} = 0.31 \pm 3\%$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $|\Delta m^2_{31}| = 2.2\text{x}10^{-3} \pm 3\%$

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

Majorana CPV Phases and $|<m>|$

IH spectrum: $m_{\text{min}} < 0.01$ eV, $\sin^2\theta -$ negligible

$$\sqrt{\Delta m^2_{\text{atm}}}|\cos 2\theta_\odot| \leq |<m>| \leq \sqrt{\Delta m^2_{\text{atm}}}.$$

"Just CP-violating" region:

$$(|<m>|_{\text{exp}})_{\text{MAX}} < \sqrt{(\Delta m^2_{\text{atm}})_{\text{MIN}}} \ ,$$

$$(|<m>|_{\text{exp}})_{\text{MIN}} > \sqrt{(\Delta m^2_{\text{atm}})_{\text{MAX}}}\,(\cos 2\theta_\odot)_{\text{MAX}} \ ,$$

$$|<m>| = \zeta\ ((|<m>|_{\text{exp}})_{\text{MIN}} \pm \Delta)\ , \quad \zeta \geq 1$$

Necessary condition for establishing CP-violation:

$$1 \leq \zeta < \frac{\sqrt{(\Delta m^2_{\text{atm}})_{\text{MIN}}}}{\sqrt{(\Delta m^2_{\text{atm}})_{\text{MAX}}}\,(\cos 2\theta_\odot)_{\text{MAX}} + 2\Delta} \simeq \frac{1}{(\cos 2\theta_\odot)_{\text{MAX}}}$$

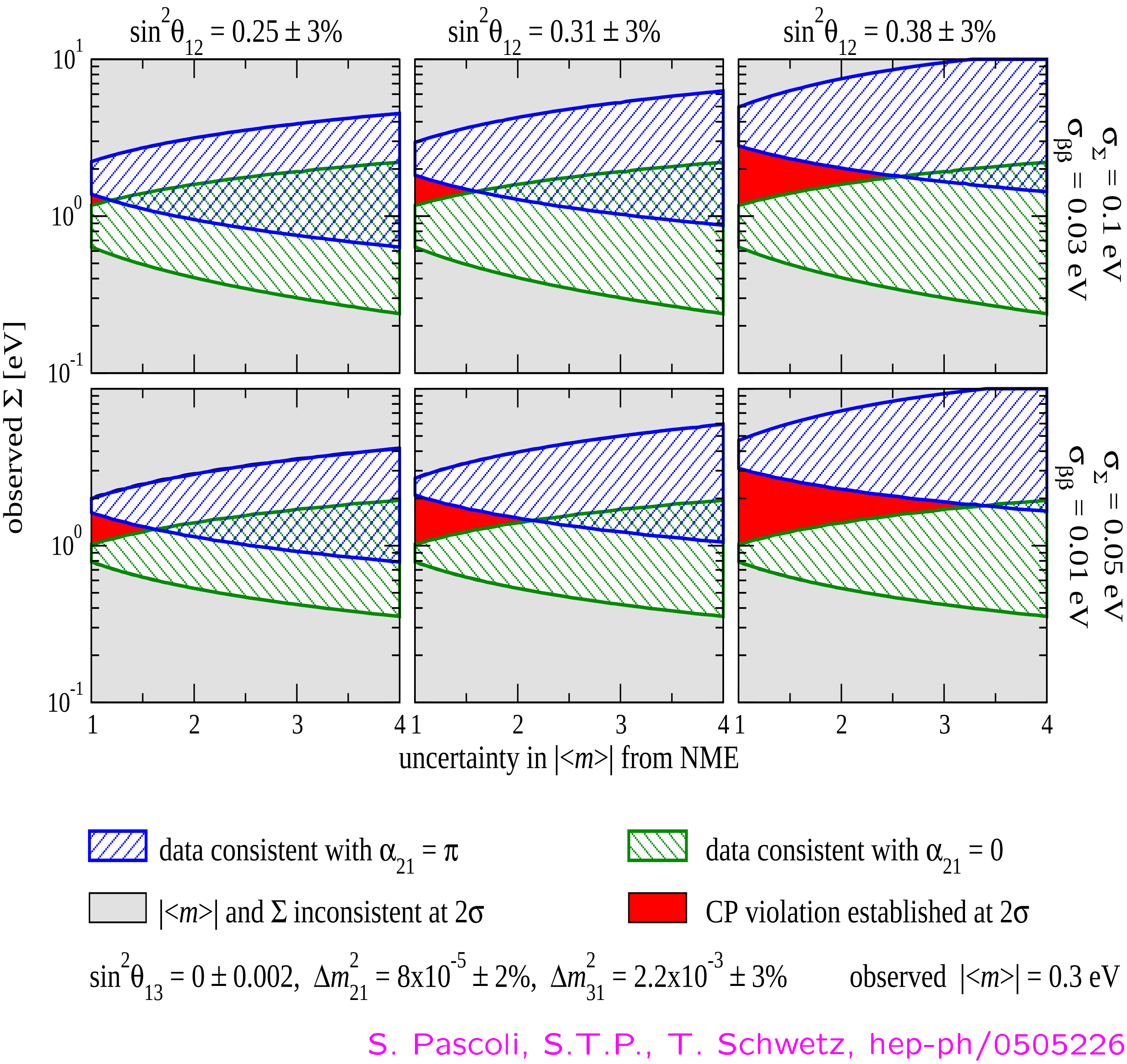
QD spectrum, $m_{1,2,3} \simeq m_0 \gtrsim 0.20$ eV, $\sin^2\theta_{13}$ negligible - similar condition: $\Delta m^2_{\text{atm}} \to m^2_0$.

CPV can be established provided

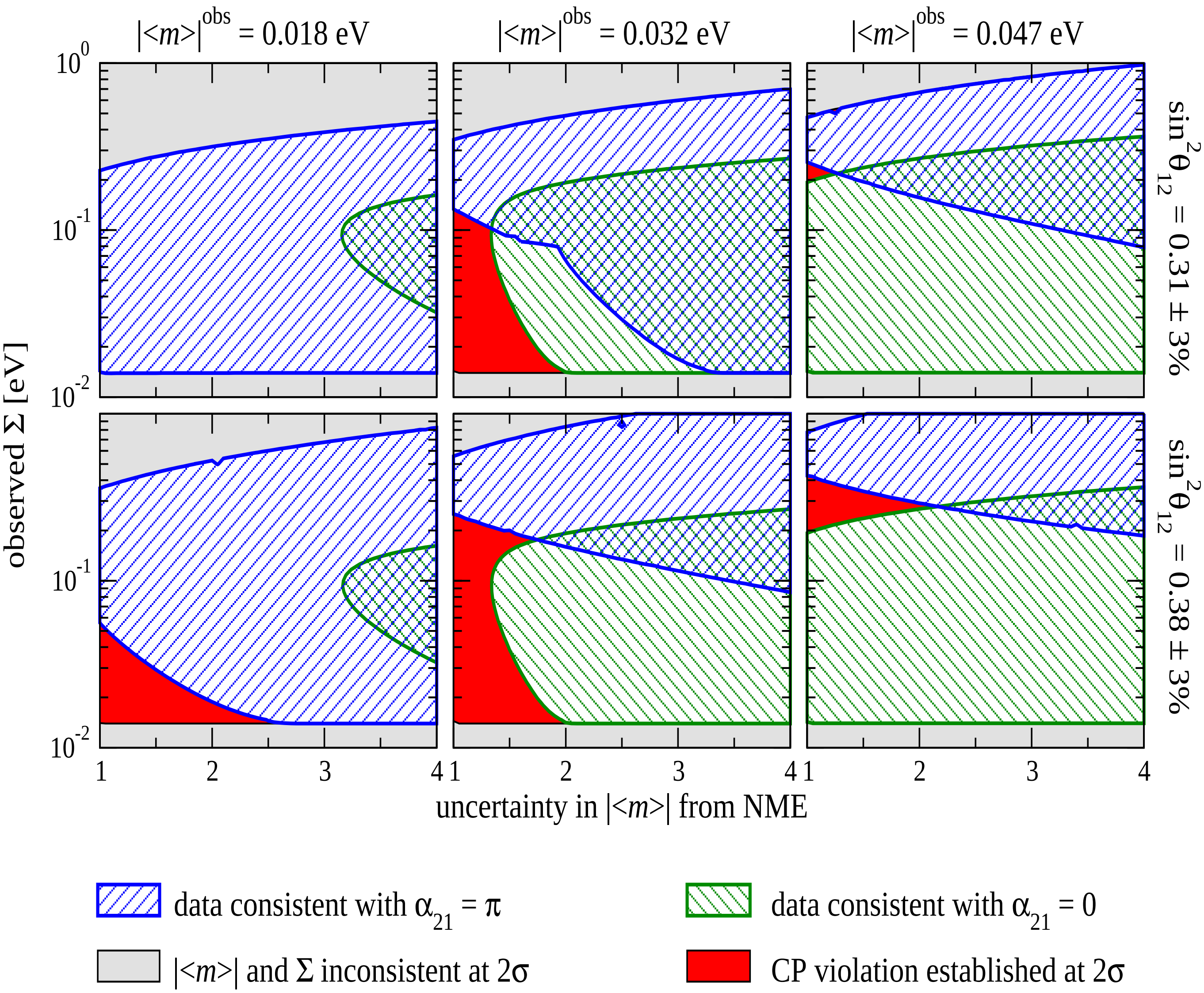
- $|<m>|$ measured with $\Delta \lesssim 15\%$;
- Δm^2_{atm} (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi < 2$ ($\xi \lesssim 1.5$) ;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2\theta_\odot \gtrsim 0.44$.

S. Pascoli, S.T.P., W. Rodejohann, 2002
S. Pascoli, S.T.P., L. Wolfenstein, 2002

"No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$-decay"
V. Barger *et al.*, 2002

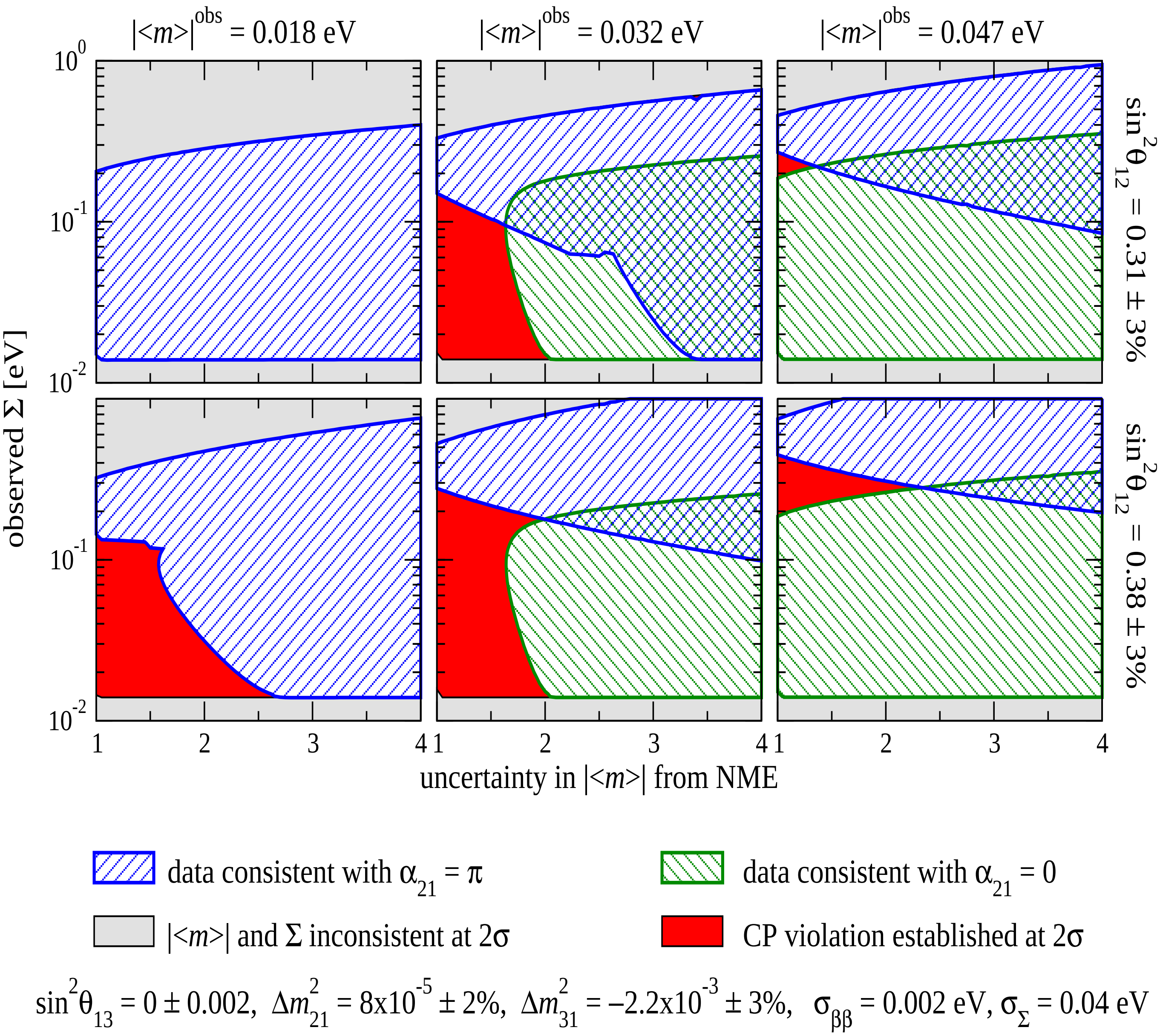


sin²θ12 = 0.25 ± 3%
sin²θ12 = 0.31 ± 3%
sin²θ12 = 0.38 ± 3%
σΣ = 0.1 eV
σββ = 0.03 eV
σΣ = 0.05 eV
σββ = 0.01 eV
observed Σ [eV]
uncertainty in |<m>| from NME
data consistent with α21 = π
data consistent with α21 = 0
|<m>| and Σ inconsistent at 2σ
CP violation established at 2σ
sin²θ13 = 0 ± 0.002, Δm²21 = 8x10^-5 ± 2%, Δm²31 = 2.2x10^-3 ± 3%
observed |<m>| = 0.3 eV
S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226



$\sin^2\theta_{13} = 0 \pm 0.002$, $\Delta m^2_{21} = 8\text{x}10^{-5} \pm 2\%$, $\Delta m^2_{31} = -2.2\text{x}10^{-3} \pm 3\%$, $\sigma_{\beta\beta} = 0.004$ eV, $\sigma_{\Sigma} = 0.04$ eV

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226



S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

On the NME Uncertainties

The $(\beta\beta)_{0\nu}$-decay half-life

$$(T^{0\,\nu}_{1/2}(A,Z))^{-1} = |<m>|^{\,2}\,|M^{0\,\nu}(A,Z)|^2\,G^{0\,\nu}(E_0,Z),$$

$G^{0\nu}(E_0,Z)$, E_0 - known phase-space factor and energy release.

If we use a model M of the calculation of NME,

$$|<m>|^{\,2}_{M}(A,Z) = \frac{1}{T^{0\,\nu}_{1/2}(A,Z)\,|M^{0\,\nu}_{M}(A,Z)|^2\,G^{0\,\nu}(E_0,Z)}.$$

Suppose $(\beta\beta)_{0\nu}$−decay of several nuclei is observed.

$|<m>|$ cannot depend on parent nucleus (A_j, Z_j).

If the light Majorana ν-exchange - dominant mechanism of $(\beta\beta)_{0\nu}$-decay, **model M for NME can be correct only if**

$$|<m>|^{\,2}_{M}(A_1,Z_1) \simeq |<m>|^{\,2}_{M}(A_2,Z_2) = ...$$

For different models and the same nucleus (A,Z),

$$|<m>|^{\,2}_{M_1}(A,Z)\;|M^{0\nu}_{M_1}(A,Z)|^2 = |<m>|^{\,2}_{M_2}(A,Z)\;|M^{0\nu}_{M_2}(A,Z)|^2 = ...,$$

$$|<m>|^{\,2}_{M_2}(A,Z) = \eta^{M_2;M_1}(A,Z)\,|<m>|^{\,2}_{M_1}(A,Z)\;,$$

$$\eta^{M_2;M_1}(A,Z) = \frac{|M^{0\,\nu}_{M_1}(A,Z)|^2}{|M^{0\,\nu}_{M_2}(A,Z)|^2}\;.$$

Nucleus	$\eta^{M_2;M_1}$	$\eta^{M_3;M_1}$	$\eta^{M_2;M_3}$
^{76}Ge	0.37	0.19	1.93
^{82}Se	—	0.38	—
^{100}Mo	—	—	6.56
^{130}Te	0.74	0.10	7.32
^{136}Xe	0.53	0.02	22.42

M_1 (SM): E. Caurier et al., 1999; M_2 (QRPA): V. Rodin et al., 2003; M_3 (QRPA): O. Civatarese and J. Suhonen, 2003.

The observation of $(\beta\beta)_{0\nu}$-decay of at least 3 nuclei would be important for the solution of the problem of NME.

Table 2 suggests: ^{76}Ge, ^{130}Te, ^{136}Xe .

If for some model M

$$|<m>|^2_M(A_1, Z_1) \simeq |<m>|^2_M(A_2, Z_2) = ... \equiv |<m>|^2_0 ,$$

$|<m>|_0$ - the true value (most likely).

Strong dependence of NME on (A, Z) - crucial for the test.

S. M. Bilenky, S.T.P., 2004

Encouraging results on the problem of calculating the NME ($\xi \lesssim 1.5$) have been obtained recently in

V. A. Rodin *et al.*, nucl-th/0503063

$\nu_\odot$, $\Delta m^2_{\rm atm}$, CHOOZ Data:

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{6}$, $\theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}$, $\theta_{13} < \frac{\pi}{12}$

$$U = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \epsilon \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (6)$$

Very different from the CKM-matrix!

- $\cos\theta_{12} \cong \cos(\frac{\pi}{4} - \frac{\pi}{12}) = \frac{1}{\sqrt{2}}(1+\lambda)$, $\sin\theta_{12} \cong \frac{1}{\sqrt{2}}(1-\lambda)$,
- $\lambda \cong (0.20 - 0.25)$: $\theta_\odot + \theta_{\rm c} = \pi/4$?

Natural Possibility:

$$U = U^\dagger_{\rm lep}(\lambda)\; U_{\rm bimax} \quad (7)$$

with

$$U_{\rm bimax} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (8)$$

- $U^\dagger_{\rm lep}(\lambda)$ - from diagonalization of the l^- mass matrix,
- $U_{\rm bimax}$ - from diagonalization of the $\nu-$mass matrix

Further, $\Delta m^2_\odot \ll |\Delta m^2_{\rm atm}|$.

- $U_{\rm bimax}$ can be associated with a symmetry:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

This symmetry cannot be exact.

The Case of CP Nonconservation

$M_{\text{lep}} = U_L^\dagger \, m_{\text{lep}}^{\text{diag}} \, U_R$, $M_\nu = U_\nu^T \, m_\nu^{\text{diag}} \, U_\nu$; $\quad U = e^{i\Phi} \, P \, \tilde{U} \, Q$

$U_{\text{PMNS}} = U_{\text{L}}^\dagger \, U_\nu = \tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$

- $\tilde{U}_{\text{lep}}$, $\tilde{U}_\nu$ - CKM-like: (3+3) angles, (1+1) CPVP
- $P_\nu = \text{diag}(1, e^{i\phi}, e^{i\omega})$, $Q_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$: 4 CPVP

U_{PMNS}: 3 angles, 3 CPVP

$\tilde{U}_{\text{lep}}^\dagger \; P_\nu \; \tilde{U}_\nu \; Q_\nu$: 6 angles, 6 CPVP; textures, symmetries

Suppose $\tilde{U}_\nu$ - bimaximal (real) and arises from

$$M_\nu = \frac{m}{\sqrt{2}} \begin{pmatrix} 0 & e^{-i\alpha'} & e^{-i\beta'} \\ e^{-i\alpha'} & 0 & \epsilon \; e^{-i\gamma'} \\ e^{-i\beta'} & \epsilon \; e^{-i\gamma'} & 0 \end{pmatrix} ,$$

α', β', γ' - phases, $\epsilon \ll 1$

$\Delta m^2_{\text{atm}} \cong m^2$, $\Delta m^2_\odot \cong \sqrt{2} \epsilon \Delta m^2_{\text{atm}}$, $\epsilon \sim 0.025$, IH $\nu-$ masses

In the limit $\epsilon = 0$ and $U_{\text{lep}} = 1$,

$L' = L_e - L_\mu - L_\tau$ is conserved.

For $U_{\text{lep}} \neq 1$, $(\alpha' - \gamma')$, $(\beta' - \gamma')$ physical CPVP,

$Q_\nu = 1$, $P_\nu = \text{diag}(1, e^{i(\beta' - \gamma')}, e^{i(\alpha' - \gamma')})$

$U_{\text{PMNS}} = \tilde{U}_{\text{lep}}^\dagger P_\nu U_{\text{bimax}}$: 3 angles, 3 CPVP

P. Frampton, S.T.P., W. Rodejohann, 2004

Rephasing Invariants Associated with CPVP

Dirac phase δ:

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\}\ .$$

C. Jarlskog, 1985

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21}, α_{31}:

$$S_1 = \text{Im}\left\{U_{e1}U^*_{e3}\right\},\quad S_2 = \text{Im}\left\{U_{e2}U^*_{e3}\right\}\quad \text{(not unique)}$$

J.F. Nieves and P. Pal, 1987, 2001
G.C. Branco et al., 1986
J.A. Aguilar-Saavedra and G.C. Branco, 2000

S_1, S_2 appear in $|{<}\,m\,{>}|$ in $(\beta\beta)_{0\nu}$-decay.

In general, J_{CP}, S_1 and S_2 are independent.

However, for, e.g., all $\lambda_{ij} \lesssim \lambda$ small, in the model we are considering and to leading order in λ,

$$J_{CP} \simeq \frac{S_1}{2\sqrt{2}} \simeq \frac{S_2}{2\sqrt{2}}\ ,$$

and

$$|{<}\,m\,{>}|\ \cong \sqrt{|\Delta m^2_{\text{atm}}|}\ |\cos 2\theta_\odot + i\,8\,J_{CP}|\ .$$

P. Frampton, S.T.P., W. Rodejohann, 2004

Conclusions

$(\beta\beta)_{0\nu}$-decay experiments have remarkable physics potential:

– Can establish the Majorana nature of ν_j

– Can provide unique information on the ν mass spectrum

– Can provide unique information on the absolute scale of ν masses

– Can provide information on the Majorana CPV phases

The knowledge of the values of the relevant $(\beta\beta)_{0\nu}$–decay NME with a sufficiently small uncertainty is crucial for obtaining quantitative information on the neutrino mass and mixing parameters from a measurement of $\Gamma(\beta\beta)_{0\nu}$.