Bounds on Neutrino Mixing Angles and CP Phase in an $SO(10)$ Model with Lopsided Mass Matrices

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- General Features of Model CHA, S.M. Barr
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- Dirac Mass Matrices and Model Parameters
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General Features of the Model

- $SO(10)$ Family Symmetry

- Higgs Structure based on a minimum set of Higgs fields which solves the doublet-triplet splitting problem and stabilizes the solution.

  $\mathbf{45_H}$,

  $\mathbf{16_H}$, $\overline{\mathbf{16}}_{\mathbf{H}}$, $\mathbf{16'_H}$, $\overline{\mathbf{16}'}_{\mathbf{H}}$,

  $\mathbf{10_H}$, $\mathbf{10'_H}$, ...,

  $\mathbf{1_H}$'s,

  where

  $\langle \mathbf{45_H} \rangle_{B-L}$, $\langle 1(\mathbf{16_H}) \rangle$, $\langle 1(\overline{\mathbf{16}}_{\mathbf{H}}) \rangle$ break $SO(10) \to$ SM;

  $v_u = \langle 5(\mathbf{10_H}) \rangle$, $v_d = \langle \bar{5}(\mathbf{10_H}) \rangle \cos\gamma + \langle \bar{5}(\mathbf{16'_H}) \rangle \sin\gamma$

  break EW symmetry.

- $U(1) \times Z_2 \times Z_2$ Flavor Symmetry exhibited by the Higgs superpotential and adopted for the model.

- Matter Superfields

  $\mathbf{16_1}$, $\mathbf{16_2}$, $\mathbf{16_3}$; $\mathbf{16}$, $\overline{\mathbf{16}}$, $\mathbf{16'}$, $\overline{\mathbf{16}'}$

  $\mathbf{10_1}$, $\mathbf{10_2}$

  $\mathbf{1}$'s, ...

  where all but the $\mathbf{16_i}$, $i = 1, 2, 3$ get superheavy and are integrated out.

- Structure of Dirac Mass Matrices then determined by Higgs and Matter superpotentials and Froggatt-Nielsen diagrams.

Dirac Mass Matrices

In the convention that the left-handed fields appear on the left and the left-handed conjugate fields on the right:

$$M_U = \begin{pmatrix} \eta & \delta_N & \delta'_N \\ \delta_N & 0 & -\epsilon/3 \\ \delta'_N & \epsilon/3 & 1 \end{pmatrix} m_U, \quad M_D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon/3 \\ \delta' e^{i\phi} & \sigma + \epsilon/3 & 1 \end{pmatrix} m_D,$$

$$M_N = \begin{pmatrix} \eta & \delta_N & \delta'_N \\ \delta_N & 0 & \epsilon \\ \delta'_N & -\epsilon & 1 \end{pmatrix} m_U, \quad M_L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon \\ \delta' e^{i\phi} & -\epsilon & 1 \end{pmatrix} m_D,$$

where $\tan\beta \simeq 5$ and

$$\begin{aligned} m_U &\simeq 113 \text{ GeV}, & m_D &\simeq 1 \text{ GeV}, \\ \sigma &= 1.83, & \epsilon &= 0.147, \\ \delta &= 0.00946, & \delta' &= 0.00827, \\ \phi &= 119.4^\circ, & \eta &= 1.1 \times 10^{-5}, \\ \delta_N &= -1.0 \times 10^{-5}, & \delta'_N &= -1.5 \times 10^{-5} \end{aligned}$$

are input parameters defined at the GUT scale to fit the low scale observables after evolution downward from Λ_G leading to:

$$\begin{aligned} m_t(m_t) &= 165 \text{ GeV}, & m_\tau &= 1.777 \text{ GeV}, \\ m_c(m_c) &= 1.23 \text{ GeV}, & m_\mu &= 105.7 \text{ MeV}, \\ m_u(1 \text{ GeV}) &= 5.5 \text{ MeV}, & m_e &= 0.55 \text{ MeV}, \\ m_b(m_b) &= 4.25 \text{ GeV}, & V_{cb} &= 0.0415, \\ m_s(1 \text{ GeV}) &= 148 \text{ MeV}, & V_{us} &= 0.2240, \\ m_d(1 \text{ MeV}) &= 7.9 \text{ MeV}, & |V_{ub}/V_{cb}| &= 0.090, \\ \delta_{CP} &= 64^\circ, & \sin 2\beta &= 0.72. \end{aligned}$$

The lopsided nature of M_D and M_L accounts for the small V_{cb} mixing and will account for the near maximal $U_{\mu 3}$ mixing.

Right-Handed Majorana Mass Matrix

- **LMA Solar Neutrino Mixing:**

$$M_R = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R,$$

  obtained from Froggatt-Nielsen diagrams leading to a 2-3 submatrix with zero determinant.

- **Seesaw Formula** with $\delta_N,\ \delta'_N$ neglected results in

$$\begin{aligned} M_\nu &= -M_N M_R^{-1} M_N^T \\ &= -\begin{pmatrix} 0 & -\epsilon/(b-a) & 0 \\ -\epsilon/(b-a) & (b^2-c^2)\epsilon^2/(b-a)^2 & b\epsilon/(b-a) \\ 0 & b\epsilon/(b-a) & 1 \end{pmatrix} \frac{m_U^2}{\Lambda_R}, \end{aligned}$$

  which exhibits a normal mass hierarchy.

- $\Lambda_R \simeq 2.85 \times 10^{14}$ GeV sets the scale for the right-handed neutrino masses and leads to an atmospheric mass squared difference, $\Delta m^2_{32} \simeq 2.4 \times 10^{-3}$ eV^2.

- $a,\ b,\ c$ are unknown complex input parameters.

Monte Carlo Searches of 3 Complex Parameters

- **General Monte Carlo Search for complex** $a,\ b,\ c$ results in upper limit for $\sin^2\theta_{13} \sim 0.01$.

- **Restricted Monte Carlo Search** with purely imaginary $a,\ b,\ c$ leads to more restricted ranges for the mixing angles and phase with larger values of baryon asymmetry, η_B, more prevalent.

- **Restricted Search with Successful Leptogenesis**
  With purely imaginary $a,\ b,\ c$ and $c = a$, the 3 mixing angles and CP phase are strongly correlated with

  $$b \simeq (1.7 \to 2.1)i,\ \ |a/b| = 0.33 \to 0.53.$$

  All acceptable points give $\eta_B \simeq (2.7 \to 6.3) \times 10^{-10}$.

- **Special Example with Successful Leptogenesis**

$$c = a = 0.5828i, \quad b = 1.7670i$$

$$\begin{aligned}
M_1 \sim M_2 &\sim 5.40 \times 10^8 \text{ GeV}, & M_3 &= 2.91 \times 10^{14} \text{ GeV}, \\
M_2 - M_1 &= 1.06 \times 10^{14} \text{ GeV}, & \Gamma_1 &= 2.07 \times 10^3 \text{ GeV},
\end{aligned}$$

$$m_1 = 3.05 \text{ meV},\ m_2 = 9.30 \text{ meV},\ m_3 = 48.9 \text{ meV},$$

$$\begin{aligned}
\Delta m^2_{32} &= 2.30 \times 10^{-3} \text{ eV}^2, & \sin^2\theta_{23} &= 0.550, \\
\Delta m^2_{21} &= 7.71 \times 10^{-5} \text{ eV}^2, & \tan^2\theta_{12} &= 0.388, \\
\delta_{CP} &= 63^\circ, & \sin^2\theta_{13} &= 0.0022,
\end{aligned}$$

$$\chi_1 = -55.3^\circ, \quad \chi_2 = 31.8^\circ, \quad \eta_B = 6.2 \times 10^{-10}$$