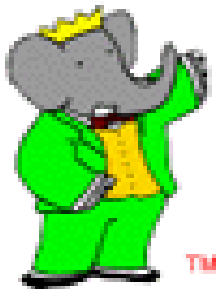




WIN`05, Delphi  
June 8, 2005

# Measurements of the angle $\alpha$ : $\rho\rho$ , $\rho\pi$ (BaBar & Belle results)



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dapnia  
SPP

cea

saclay

# Outline

- Physics motivation
- Measurement of  $\alpha$  in  $B \rightarrow \rho\rho$ .
- Measurement of  $\alpha$  in  $B \rightarrow \rho\pi$  Dalitz.
- Summary on  $\alpha$ .

# CP violation

- CP violation is explained in the Standard Model by a phase in the CKM unitary matrix.
- In the Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with  $\lambda \cong 0.22$  ,  $A \cong 0.83$

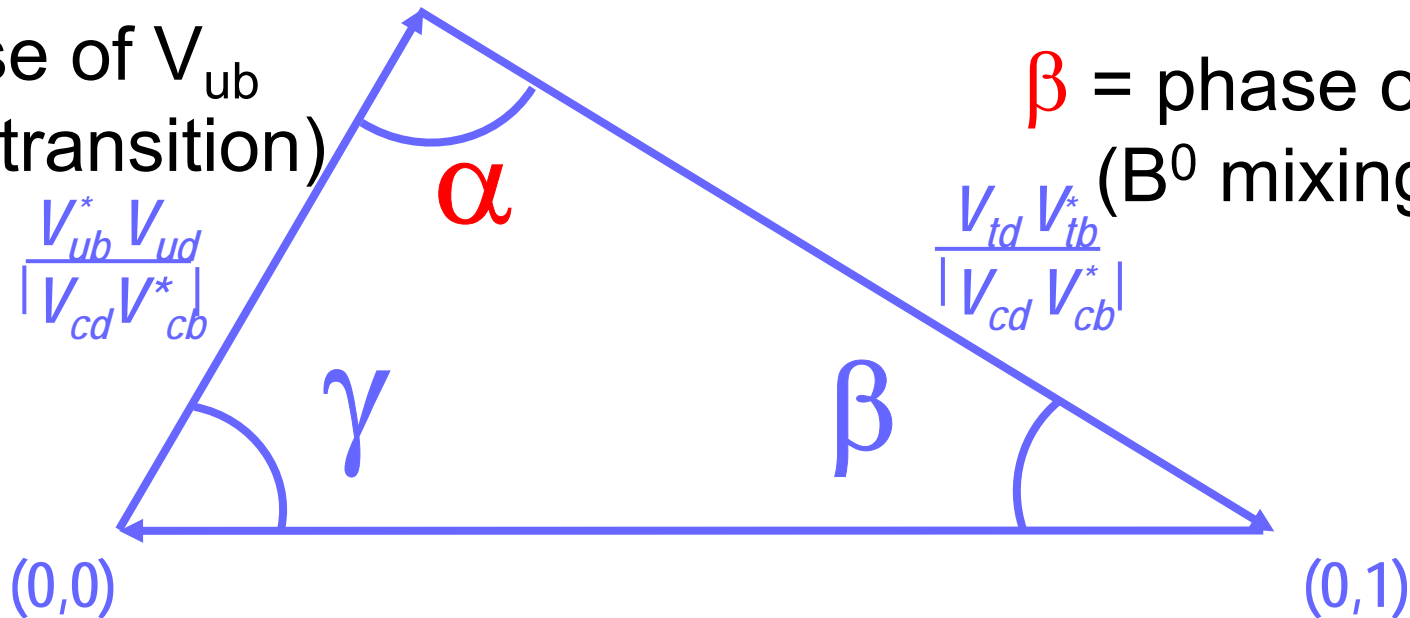
- CP violation if  $\eta \neq 0$ .

# The unitarity triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$\gamma$  = phase of  $V_{ub}$   
( $b \rightarrow u$  transition)

$\beta$  = phase of  $V_{td}$   
( $B^0$  mixing)



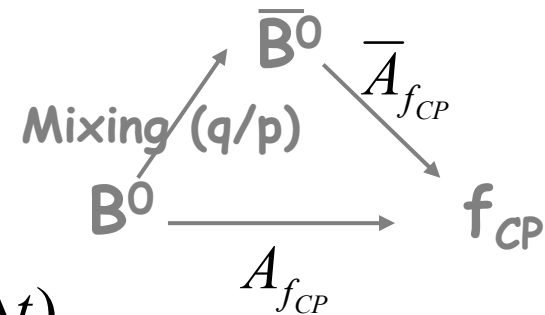
$\alpha = \pi - \beta - \gamma$  = process involving both  $B^0$  mixing and  $b \rightarrow u$  transition

# CP violation in the interference between mixing and decay

- For a CP final state  $f_{CP}$ , time-dependent asymmetry is:

$$A_{f_{CP}}(\Delta t) = \frac{\Gamma(\bar{B}^0 \rightarrow f_{CP}) - \Gamma(B^0 \rightarrow f_{CP})}{\Gamma(\bar{B}^0 \rightarrow f_{CP}) + \Gamma(B^0 \rightarrow f_{CP})}$$

$$= S_{f_{CP}} \sin(\Delta m_d \Delta t) - C_{f_{CP}} \cos(\Delta m_d \Delta t)$$



$$S_{f_{CP}} = \frac{2 \Im(\lambda_{f_{CP}})}{1 + \lambda_{f_{CP}}^2}$$

$$C_{f_{CP}} = \frac{1 - \lambda_{f_{CP}}^2}{1 + \lambda_{f_{CP}}^2}$$

$$\lambda_{f_{CP}} = \frac{q \bar{A}_{f_{CP}}}{p A_{f_{CP}}}$$

Final State Amplitudes

$S \neq 0$  : Indirect CP violation

$C \neq 0$  : Direct CP violation

from mixing

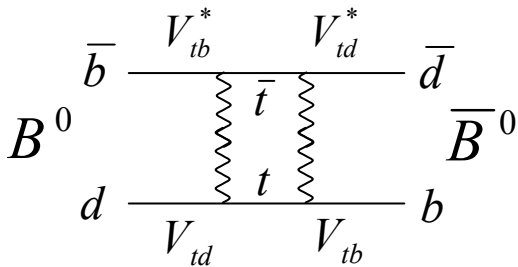
$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

# CP violation in $B^0 \rightarrow \rho^+ \rho^-$

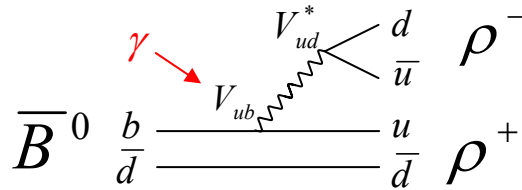
- Access to  $\alpha$  from the interference of a  $b \rightarrow u$  decay ( $\gamma$ ) with  $B^0 \bar{B}^0$  mixing ( $\beta$ ).

$B^0 \bar{B}^0$  mixing



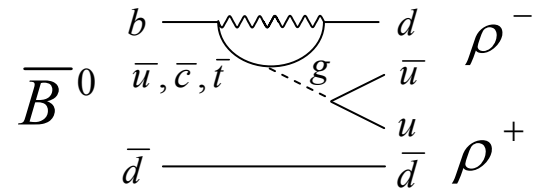
$$q/p \propto V_{tb}^* V_{td} / V_{tb} V_{td}^*$$

Tree decay



$$A \propto V_{ud}^* V_{ub}$$

Penguin decay



$$A \approx V_{td}^* V_{tb}$$

Tree only

$$\lambda_{\rho^+ \rho^-} = \frac{q}{p} \frac{\bar{A}}{A} = e^{-i2\beta} e^{-i2\gamma} = e^{i2\alpha}$$

$$S = \sin(2\alpha)$$

$$C = 0$$

Tree + Penguin

$$\lambda_{\rho^+ \rho^-} = e^{i2\alpha} \frac{T + P e^{+i\gamma} e^{i\delta}}{T + P e^{-i\gamma} e^{i\delta}}$$

$$S = \sqrt{1 - C^2} \sin(2\alpha_{\text{eff}})$$

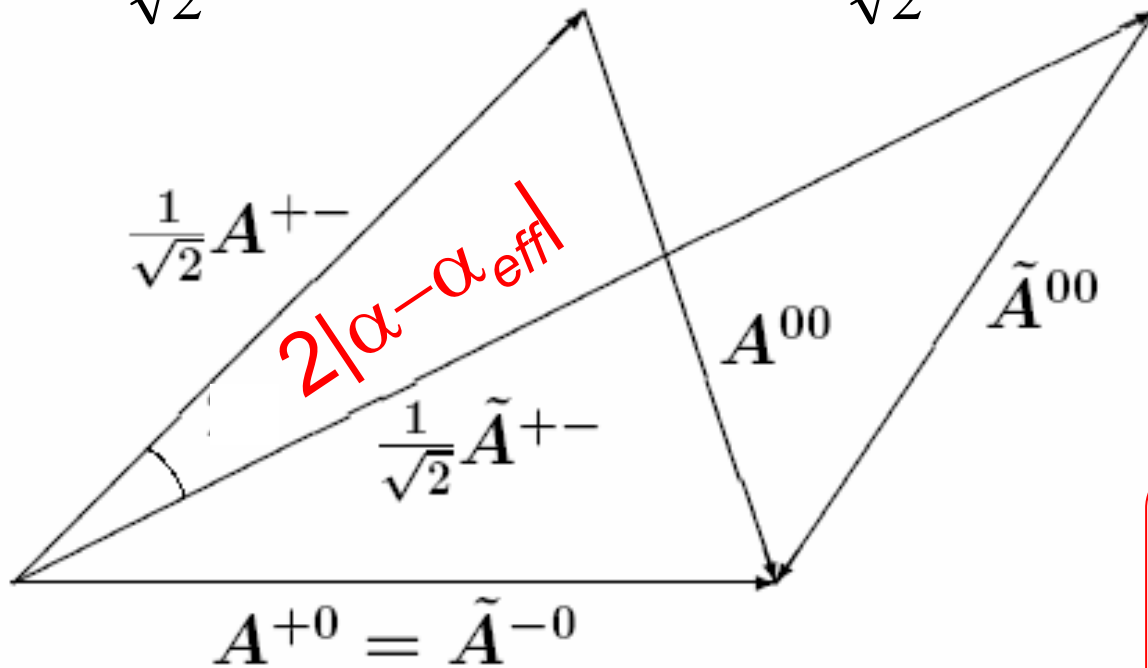
$$C \propto \sin \delta$$

How can we obtain  $\alpha$  from  $\alpha_{\text{eff}}$ ?

# Isospin analysis

- Use SU(2) to relate amplitudes of all  $\rho\rho$  modes.

$$\frac{\mathbf{A}^{+-}}{\sqrt{2}} + \mathbf{A}^{00} = \mathbf{A}^{+0} = \tilde{\mathbf{A}}^{-0} = \frac{\tilde{\mathbf{A}}^{+-}}{\sqrt{2}} + \tilde{\mathbf{A}}^{00}$$



$$\mathbf{A}^{+-} = A(B^0 \rightarrow \rho^+ \rho^-)$$

$$\tilde{\mathbf{A}}^{+-} = A(\bar{B}^0 \rightarrow \rho^+ \rho^-)$$

$$\mathbf{A}^{+0} = A(B^+ \rightarrow \rho^+ \rho^0)$$

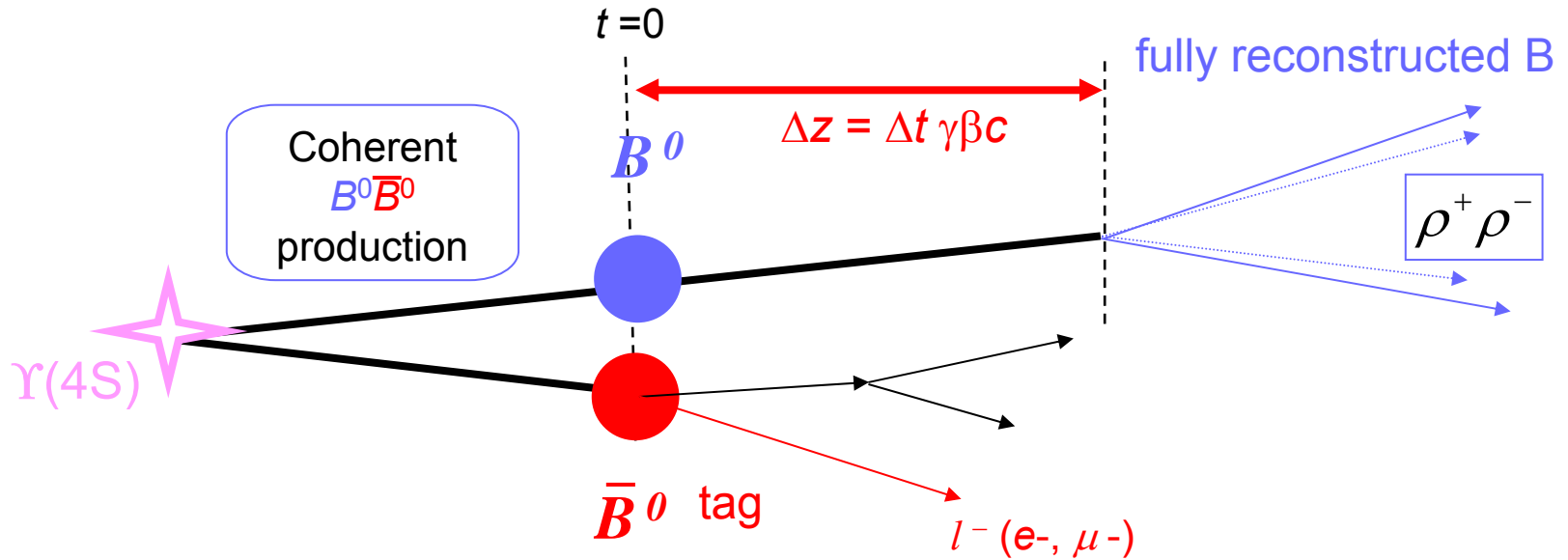
$$\mathbf{A}^{00} = A(B^0 \rightarrow \rho^0 \rho^0)$$

$$\tilde{\mathbf{A}}^{00} = A(\bar{B}^0 \rightarrow \rho^0 \rho^0)$$

Gronau, London : PRL**65**, 3381 (1990)

Small amplitudes

# Asymmetry measurement



- Exclusive B meson reconstruction.
- Time measurement:  $\Delta z \approx 250 \mu m$ ,  $\sigma_{\Delta z} \approx 170 \mu m$ .
- B-flavor tagging:  $Q = \sum \varepsilon(1-2\omega)^2 \approx 30\%$ .
  - with  $\varepsilon$  efficiency and  $\omega$  mistag rate.



# Common features of the analyses

- Kinematical signal identification with
  - Beam energy substituted mass  $m_{ES} = \sqrt{E_{beam}^{*2} - p_B^{*2}}$
  - Energy difference  $\Delta E = E_B^* - E_{beam}^*$
- Hadron ID (separation  $\pi/K$ ).
- Event-shape variables combined in a neural network (NN) or Fisher discriminant to suppress jet-like continuum event.

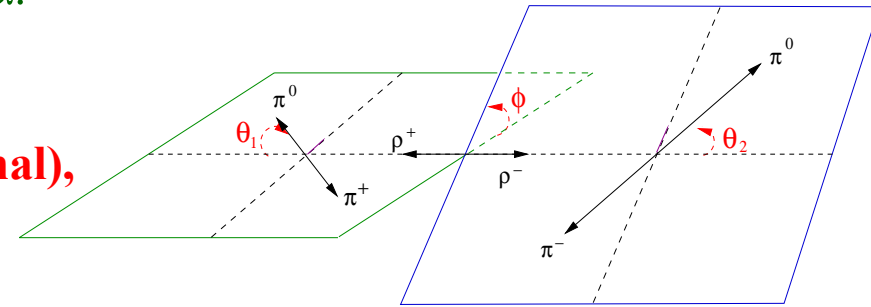
# $B^0 \rightarrow \rho^+ \rho^-$ analysis

$B \rightarrow \rho^+ \rho^-$  not historically favored for measuring  $\alpha$ :

- 2  $\pi^0$ s in the final state.
- 3 amplitudes (VV decay):  $A_0$  (CP-even longitudinal),  $A_{\parallel}$  (CP-even transverse),  $A_{\perp}$  (CP-odd transverse).

But turned out to be the best mode:

- Large branching fraction.



$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d \cos \theta_1 d \cos \theta_2} = \frac{9}{4} \left\{ \frac{1}{4} (1 - f_L) \sin^2 \theta_1 \sin^2 \theta_2 + f_L \cos^2 \theta_1 \cos^2 \theta_2 \right\}$$

$$B(B^0 \rightarrow \rho^+ \rho^-) = (30 \pm 4 \pm 5) \cdot 10^{-6} \quad (89 M_{BB})$$

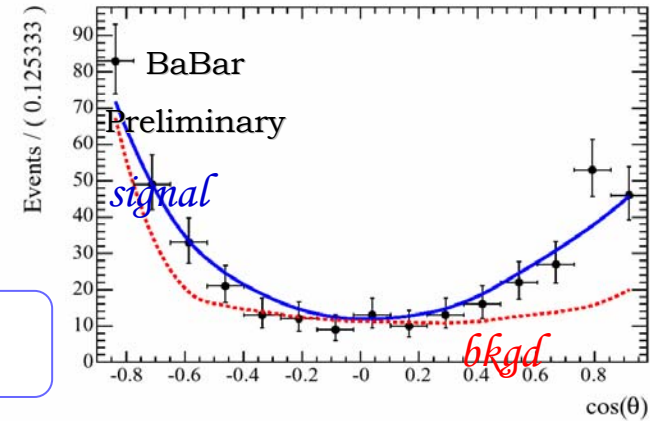
BaBar, Phys.Rev.Lett 93, 231801 (2004)

- Penguin pollution much smaller than in  $B \rightarrow \pi\pi$ .
- ~100% longitudinally polarized! Pure CP-even state.

$$f_L(\rho^+ \rho^-) = 0.978 \pm 0.014^{+0.021}_{-0.029}$$

232  $M_{BB}$

BaBar, hep-ex/050349, submitted to PRL



# Details of the $B^0 \rightarrow \rho^+ \rho^-$ analysis

- Unbinned extended **maximum likelihood fit** on a data sample of **68703 events**. 232  $M_{BB}$
- Efficiency on signal: **7.7%**.
- 8 observables:  $m_{ES}$ ,  $\Delta E$ ,  $\Delta t$ , NN,  $m_{\pi\pi}$  (x2),  $\cos\Theta^{\pi\pi}_{hel}$  (x2).
- Modelisation of signal (1% of fit sample), continuum (92% of fit sample), and 38 different modes of B-background (7% of fit sample).
- Extract **signal yield, longitudinal polarization fraction, cosine and sine coefficients**.

# $\mathcal{A}_{CP}(t)$ in $B^0 \rightarrow \rho^+ \rho^-$ decays



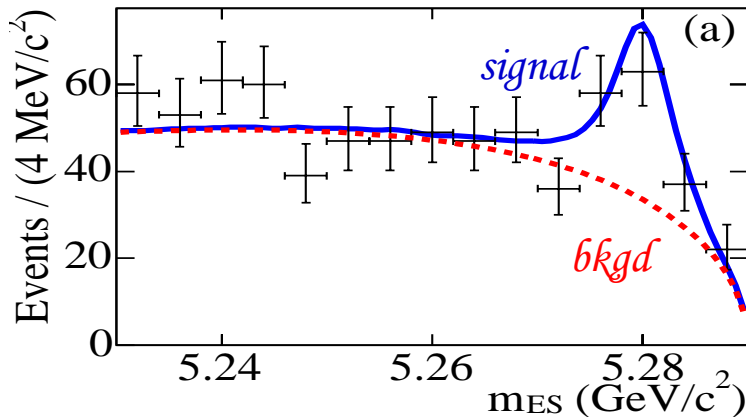
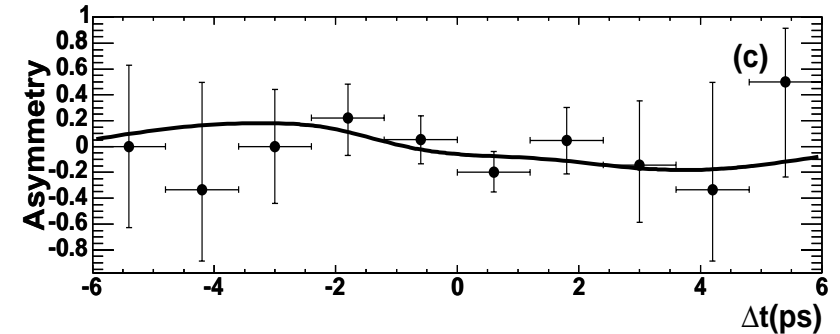
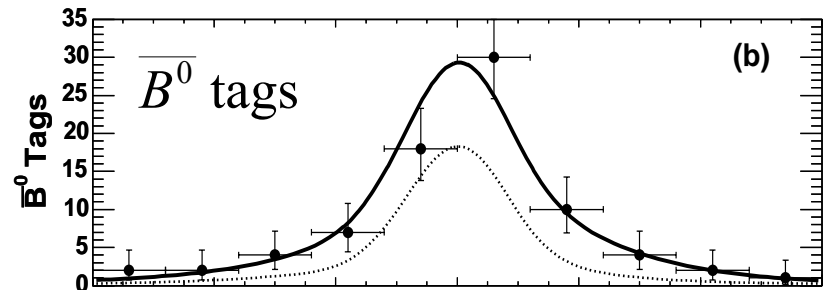
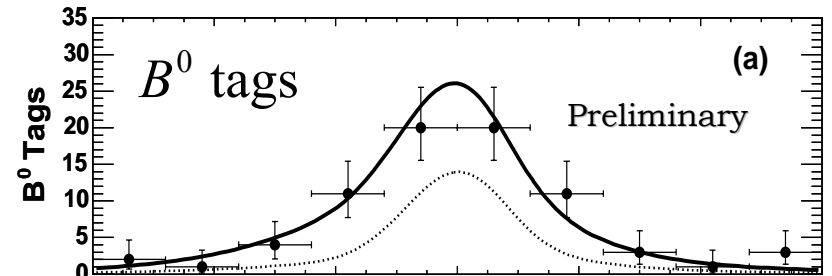
BaBar, hep-ex/0503049, submitted to PRL

232  $M_{BB}$

$$N(B \rightarrow \rho^+ \rho^-) = 617 \pm 52$$

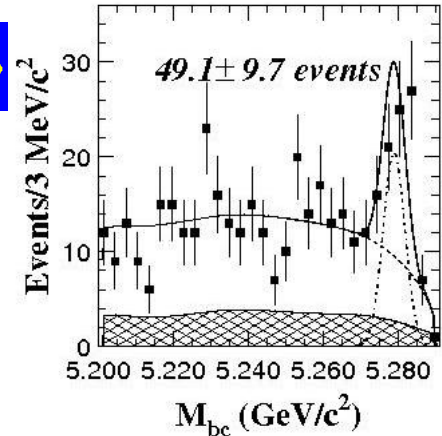
$$S_{long} = -0.33 \pm 0.24^{+0.08}_{-0.14}$$

$$C_{long} = -0.03 \pm 0.18 \pm 0.09$$



# $B^+ \rightarrow \rho^+ \rho^0$ analysis

- For isospin analysis, need other  $B \rightarrow \rho\rho$  rates.
- $B^+ \rightarrow \rho^+ \rho^0$  was measured two years ago by both BaBar and Belle.



89  $M_{BB}$

BaBar :

$$B(B^+ \rightarrow \rho^+ \rho^0) = (22.5^{+5.7}_{-5.4} \pm 5.8)10^{-6}$$

$$f_L(\rho^+ \rho^0) = 0.97^{+0.003}_{-0.007} \pm 0.04$$



Phys.Rev.Lett 91, 171802 (2003)

85  $M_{BB}$

Belle :

$$B(B^+ \rightarrow \rho^+ \rho^0) = (31.7 \pm 7.1^{+3.8}_{-6.7})10^{-6}$$

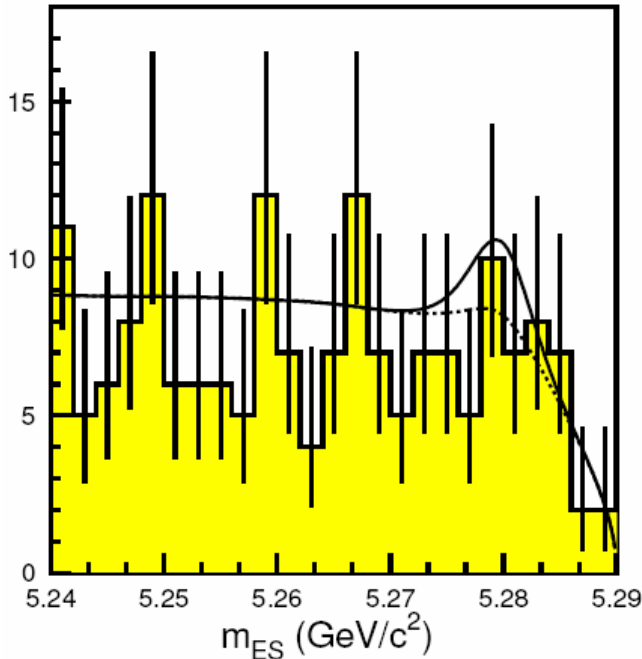
$$f_L(\rho^+ \rho^0) = 0.95 \pm 0.11 \pm 0.02$$



Phys.Rev.Lett 91, 221801 (2003)

$$\text{World average : } B(B^+ \rightarrow \rho^+ \rho^0) = (26.4 \pm 6.4)10^{-6} \quad f_L(\rho^+ \rho^0) = 0.96^{+0.005}_{-0.007}$$

# $B^0 \rightarrow \rho^0 \rho^0$ analysis



BaBar, Phys.Rev.Lett 94,  
131801 (2005)

( $227 M_{B\bar{B}}$ )

$$N(B^0 \rightarrow \rho^0 \rho^0) = 33_{-20}^{+22} \pm 12$$

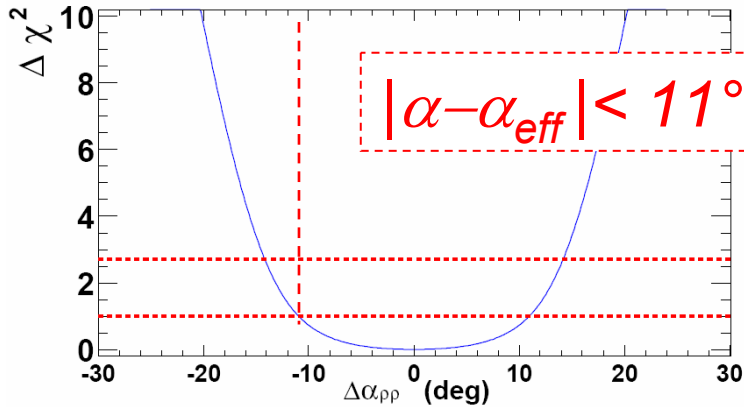
$$\text{Rec. Eff.} = 27\%$$

$$B(B^0 \rightarrow \rho^0 \rho^0) = (0.54_{-0.32}^{+0.36} \pm 0.19) \cdot 10^{-6}$$

$$< 1.1 \cdot 10^{-6} \quad 90\% \text{ C.L.}$$

- Limiting factor in the isospin analysis.
- Tree is color suppressed.
- No significant signal:
  - Penguin are smalls.
- Dominant systematic comes from the potential interference from  $B \rightarrow a_1^\pm \pi^\pm$  (~22%).

# Isospin analysis with $B \rightarrow \rho\rho$



$BF(B^0 \rightarrow \rho^+ \rho^-) = (30 \pm 4 \pm 5) \times 10^{-6}$   
 $BF(B^+ \rightarrow \rho^+ \rho^0) = (26.4 \pm 6.4) \times 10^{-6}$   
 $BF(B^0 \rightarrow \rho^0 \rho^0) < 1.1 \times 10^{-6} \text{ (90\% CL)}$

Phys.Rev.Lett 93, 231801 (2004)

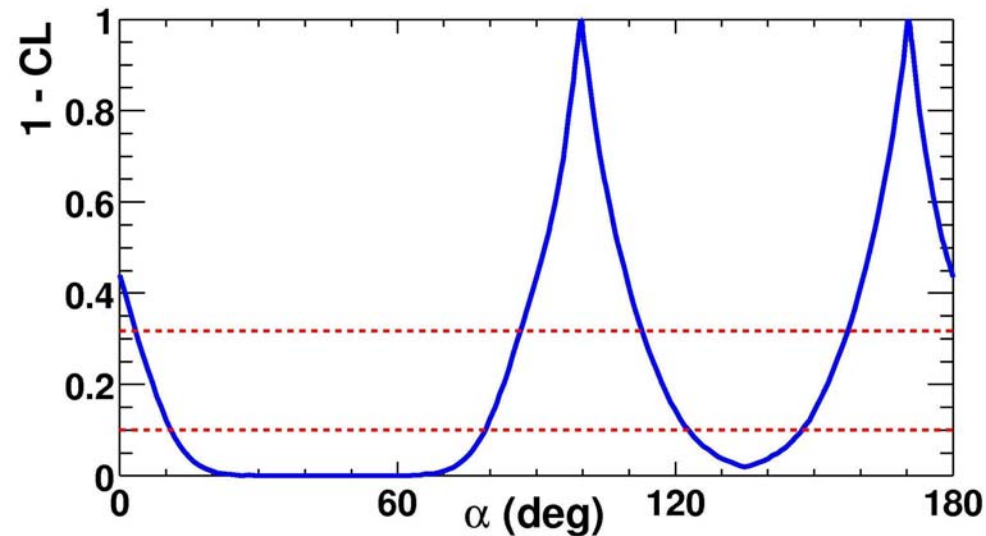
Phys.Rev.Lett 91, 171802 (2003)

Phys.Rev.Lett 91, 221801 (2003)

Phys.Rev.Lett 94, 131801 (2005)

Error dominated by  $\rho^0\rho^0$  measurement

$\alpha = 100 \pm 13^\circ$   
 90%CL range :  $[79^\circ, 123^\circ]$



BaBar, hep-ex/050349, submitted to PRL



# $B^0 \rightarrow (\rho\pi)^0$ analysis

- Unlike  $\rho^+\rho^-$ ,  $\rho^+\pi^-$  is not a CP eigenstate

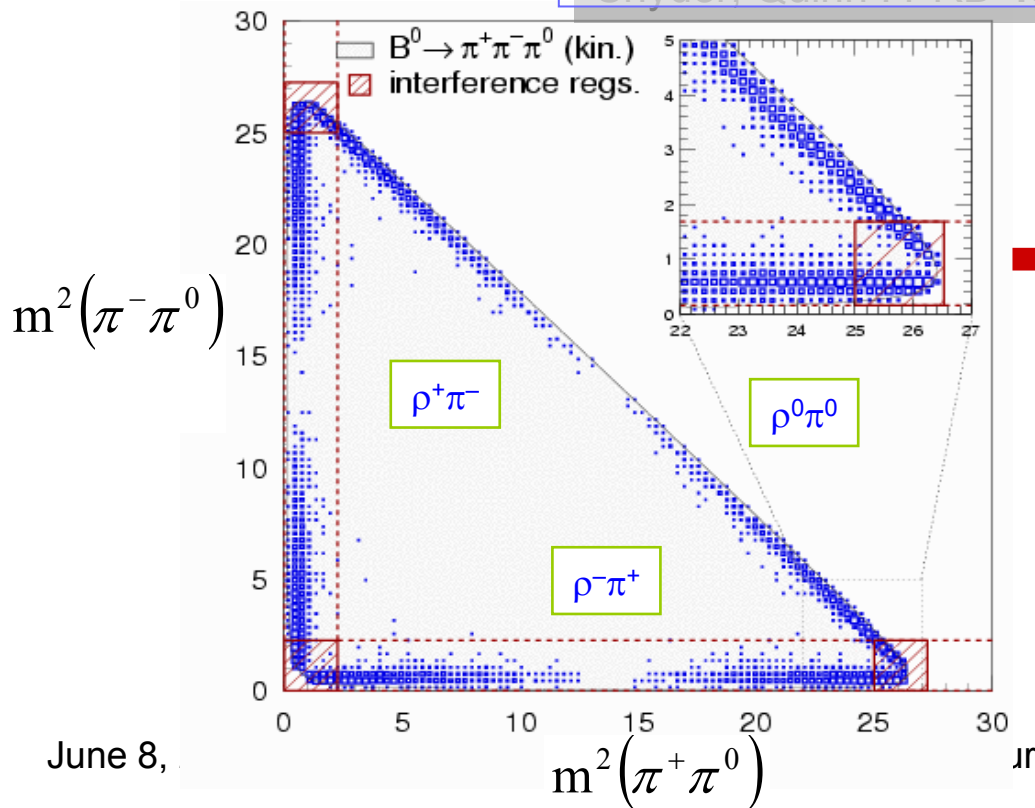
- Must consider 4 configurations

- Equivalent "isospin analysis" not viable.

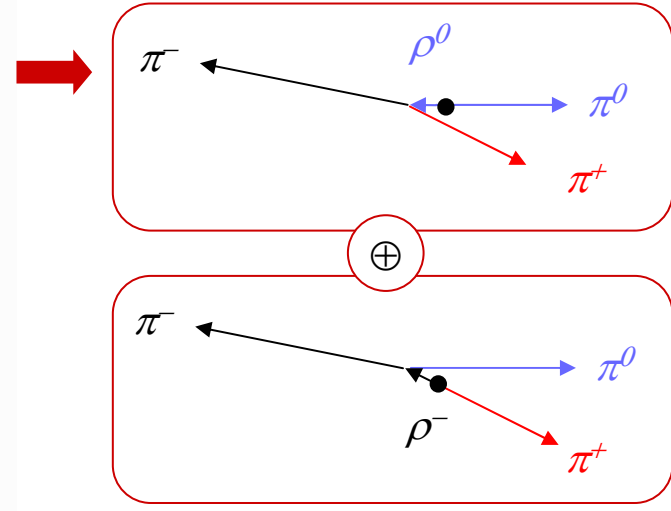
$$\begin{array}{ll} B^0 \rightarrow \rho^+\pi^- & \bar{B}^0 \rightarrow \rho^+\pi^- \\ \bar{B}^0 \rightarrow \rho^-\pi^+ & B^0 \rightarrow \rho^-\pi^+ \end{array}$$

- However, a full time-dependent Dalitz plot analysis can constrain  $\alpha$ .

Snyder, Quinn : PRD 48, 2139 (1993)



Interference at equal masses-squared gives information on strong phases between resonances





# Time-dependent Dalitz analysis

- Extract  $\alpha$  and strong phases using interferences between amplitudes of  $B \rightarrow \pi^+ \pi^- \pi^0$  decay.

$$|\mathcal{A}_{3\pi}^{\pm}(\Delta t)|^2 = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left[ |\mathcal{A}_{3\pi}|^2 + |\bar{\mathcal{A}}_{3\pi}|^2 \mp \left( |\mathcal{A}_{3\pi}|^2 - |\bar{\mathcal{A}}_{3\pi}|^2 \right) \cos(\Delta m_d \Delta t) \right. \\ \left. \pm 2\text{Im} \left[ \bar{\mathcal{A}}_{3\pi} \mathcal{A}_{3\pi}^* \right] \sin(\Delta m_d \Delta t) \right],$$

$\mathcal{A}_{3\pi}^+$  for  $B^0$   
 $\mathcal{A}_{3\pi}^-$  for  $\bar{B}^0$

- Assuming amplitude is dominated by  $\rho^+, \rho^-$  and  $\rho^0$  resonances

$$\begin{aligned} A_{3\pi} &= f_+ A^+ + f_- A^- + f_0 A^0 \\ \bar{A}_{3\pi} &= f_+ \bar{A}^+ + f_- \bar{A}^- + f_0 \bar{A}^0 \end{aligned} \quad \begin{array}{l} \text{script } \{+, -, 0\} \\ \text{refers to } \{\rho^+, \rho^-, \rho^0\} \end{array}$$

- The "f"s are functions of the Dalitz-plot and describe the kinematics of  $B \rightarrow \rho \pi$  ( $S \rightarrow VS$ ).
- The "A"s are the complex amplitudes containing weak and strong phases. They are independent of the Dalitz variables.

# Result with $B \rightarrow \pi^+ \pi^- \pi^0$

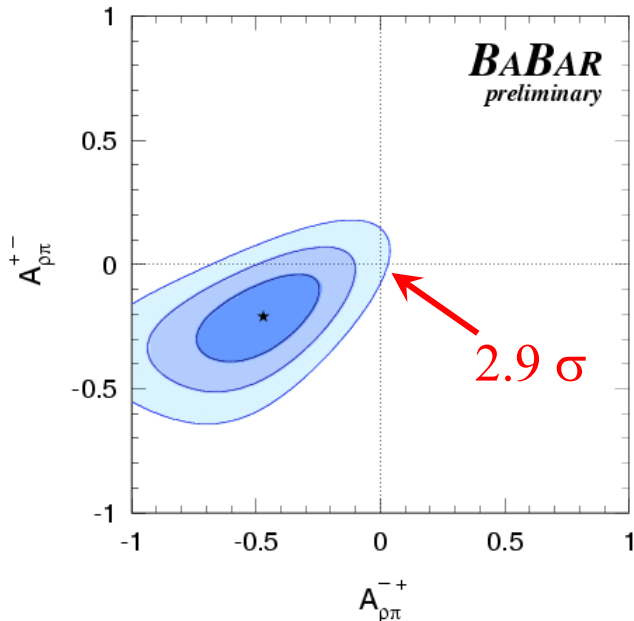
- Hint of direct  $CP$ -violation

$$A_{\rho\pi}^{+-} \cong \left[ B \begin{array}{c} \swarrow \rho^+ \\ \longrightarrow \pi^- \end{array} \right] - \left[ B \begin{array}{c} \swarrow \rho^- \\ \longrightarrow \pi^+ \end{array} \right]$$

$$= -0.21 \pm 0.11 \pm 0.04$$

$$A_{\rho\pi}^{-+} \cong \left[ B \begin{array}{c} \swarrow \pi^+ \\ \longrightarrow \rho^- \end{array} \right] - \left[ B \begin{array}{c} \swarrow \pi^- \\ \longrightarrow \rho^+ \end{array} \right]$$

$$= -0.47^{+0.14}_{-0.15} \pm 0.06$$



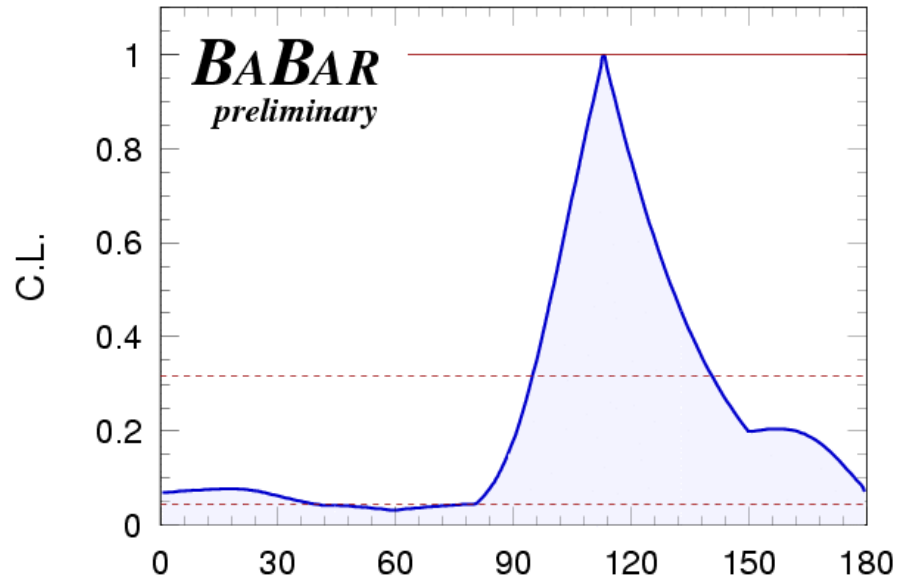
213  $M_{BB}$

- Likelihood scan of  $\alpha$  using:

$$A^\kappa = T^\kappa e^{-i\alpha} + P^\kappa$$

$$\bar{A}^\kappa = T^{\bar{\kappa}} e^{+i\alpha} + P^{\bar{\kappa}}$$

$\kappa \in \{+, -, 0\}$   
 $T$  = tree amp.  
 $P$  = penguin



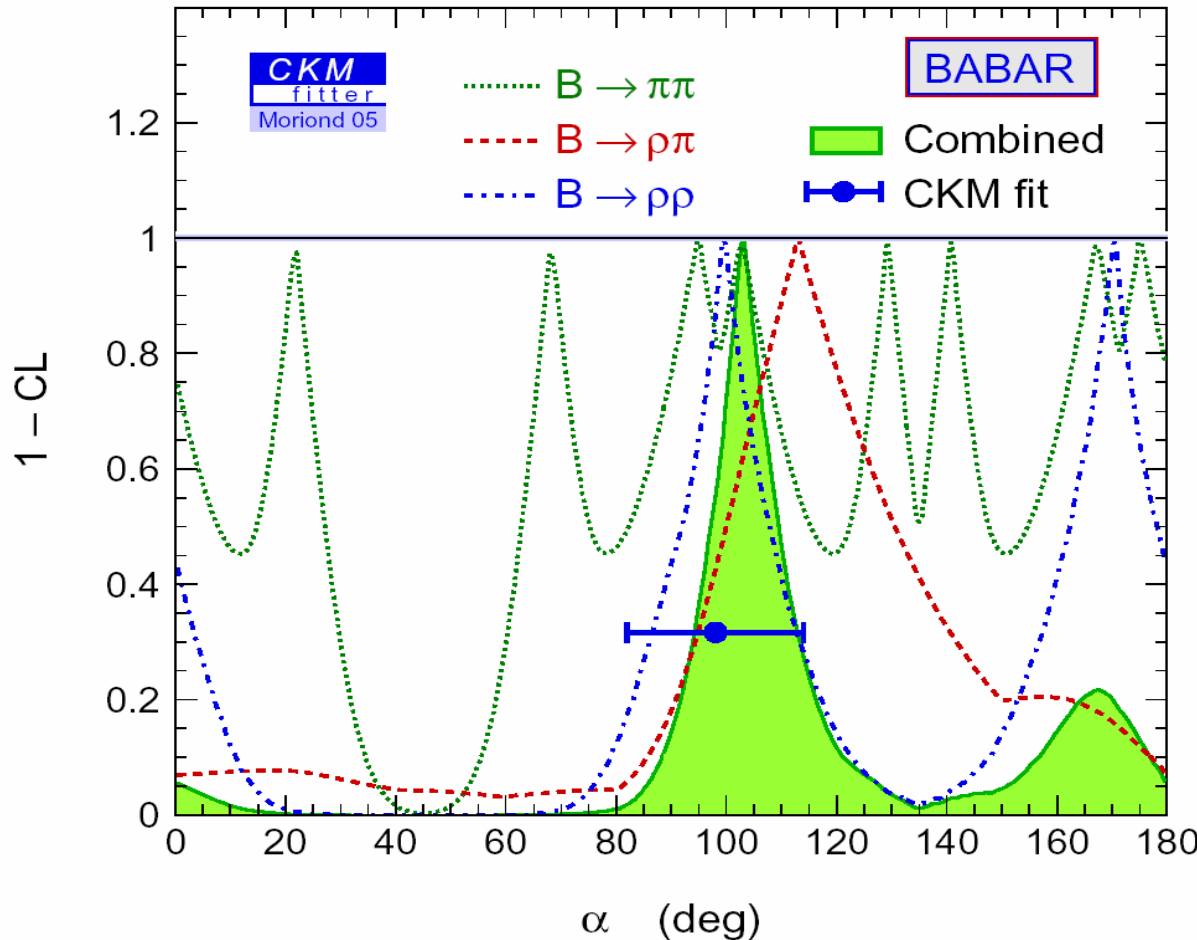
$$\alpha = \left[ 113^{+27}_{-17} (stat.) \pm 6 (syst.) \right]^\circ$$

Mirror solution not shown

# Combined $\alpha$ measurement

- The best individual measurement comes from  $\rho\rho$ .
- Mirror solutions are disfavored, thanks to  $\rho\pi$ .
- Good agreement with global CKM fit.
- Combined value:

$$\alpha = (103_{-9}^{+10})^\circ$$



<http://ckmfitter.in2p3.fr>

# Summary

- CP violation has entered a phase of precision measurements thanks to the B-factories.
- The angle  $\alpha$  of the unitarity triangle has been measured with an uncertainty of  $\sim 10^\circ$ .

$$\alpha = (103_{-9}^{+10})^\circ$$

- Will still improve.

