Theory Status of the inclusive B Decays $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s l^+ l^-$

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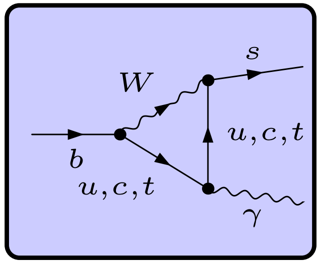
- Inclusive Decays and FCNC
- Resummation of Logs
- $\bar{B} \to X_s \gamma$ and m_c
- Towards NNLO
- Improvements for $\bar{B} \to X_s l^+ l^-$
- Forward-Backward Asymmetry

WIN 2005

$b \to s\gamma$ as a typical FCNC Decay



top loop $\propto V_{tb}V_{ts}^* = \mathcal{O}(\lambda^2) \to -100\%$

charm loop $\propto V_{cb}V_{cs}^* \approx -V_{tb}V_{ts}^* \to +200\%$

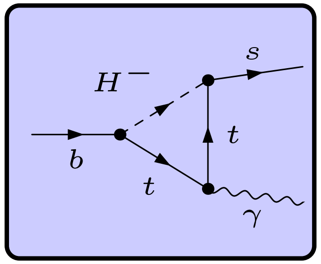
up loop $\propto V_{ub}V_{us}^* = \mathcal{O}(\lambda^4) \to 0$

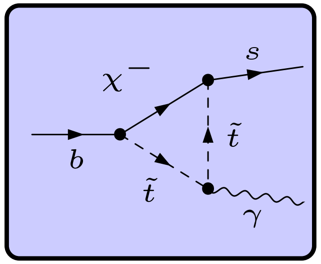
including LO QCD

In the SM forbidden at tree level & CKM suppressed

Precision test of the flavour sector

Enhanced sensitivity to new physics





- Charged Higgs contribution enhance $b \to s\gamma$
- Different new physics contributions have to cancel

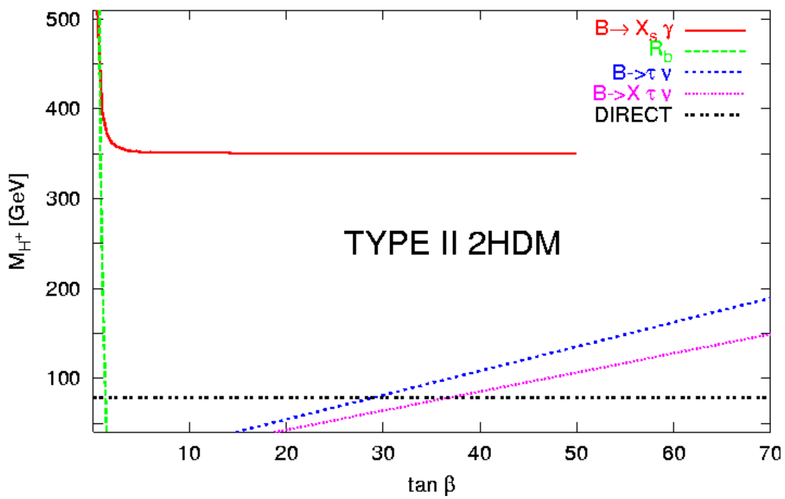
$b \to s\gamma$ Constrains New Physics

- World average [HFAG '04]

$$\mathrm{BR}_\gamma^{\mathrm{exp}} = (3.52 \pm 0.30) \times 10^{-4}$$

- NLO SM prediction [Gambino, Misiak '01]

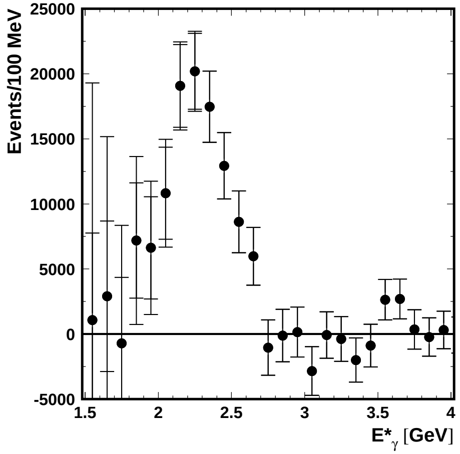
$$\mathrm{BR}_\gamma^{E_\gamma > m_b/20} = (3.70 \pm 0.30) \times 10^{-4}$$



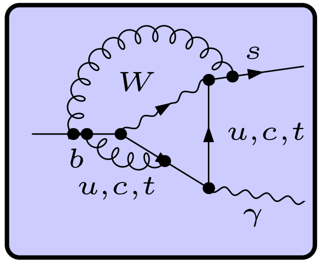
- Precision test of the flavor sector of the SM
- Large sensitivity on new physics
- Inclusive decay is theoretically clean
- Calculated up to three loops
- What a pity that there is no disagreement with the SM

Bound States and Large Logs

- We can only observe decays of bound states $\Rightarrow$ decay at parton level may not approximate the hadronic decay
- Study inclusive decay:
  $\mathrm{BR}_\gamma = (3.34 \pm 0.38) \times 10^{-4}$



- New measurement by Belle:
  $\mathrm{BR}_\gamma = (3.59 \pm 0.32^{+0.30}_{-0.31}{}^{+0.11}_{-0.07}) \times 10^{-4}$
  used $E_\gamma > 1.8\mathrm{GeV}$



For n gluons we have

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^n \frac{m_b^2}{M_W^2} (LL)$$

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^{n-1} \frac{m_b^2}{M_W^2} (NLL)$$

- Large logs $\Rightarrow$ straightforward perturbation theory unreliable
- Leading Log enhance Branching Ratio by 200%
- Use renormalisation group to resum leading and next-to-leading logs

Inclusive $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s l^+ l^-$ Decays

Sum over all X_s final states

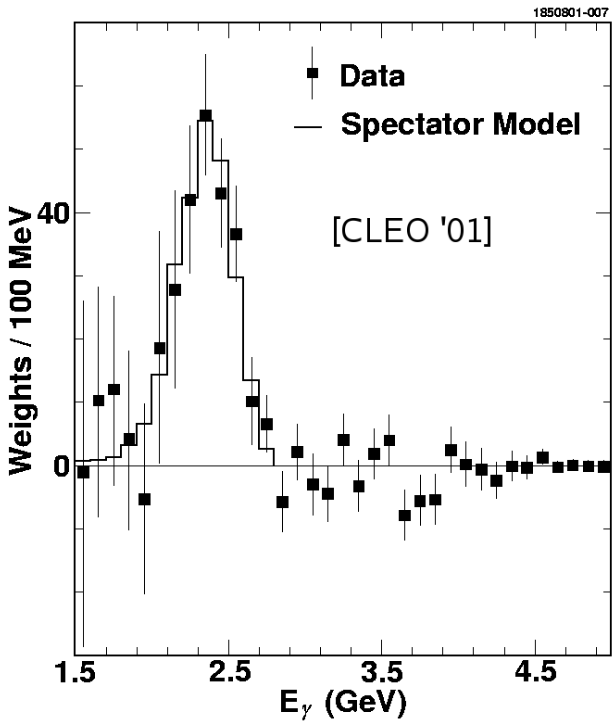
$m_b \gg \Lambda_{QCD}$ hadron binding energy

Contribution of external states drops out

- For $m_b \to \infty$ is $\Gamma[\bar{B} \to X_s\gamma] \approx \Gamma[b \to s\gamma] + \Gamma[b \to s\gamma g]^\delta + \ldots$ [Chay et al. '90, Manohar et al. '93]
- $1/m_b^2$ and $1/m_c^2$ corrections can be added systematically [Falk et al. '93, Bigi '92, Voloshin '97, Khodjamirian et al. '00]
- Treatment of $\bar{B} \to X_s l^+ l^-$ is similar to $\bar{B} \to X_s\gamma$ [Ali et al. '96, Bauer et al. '99, Chen et al. '97, Buchalla et al. '97]
- Theoretically a cut $E_\gamma^{\text{theory}} = 1.6\text{GeV}$ for $\bar{B} \to X_s\gamma$ would be preferred
- So far experiments can not go so low

Is a $E_\gamma^{\rm theory} = 1.9\rm GeV$ cut-off reliable?

- Experiments use: Cleo: $E_\gamma = 2.0\rm GeV$ Belle: $E_\gamma = 1.8\rm GeV$
- Theorists believe that for $E_\gamma < 1.9\rm GeV$ dependence on shape function disappear and OPE is reliable



- In the presence of a photon cut off there is a dependence on short scales [Neubert '04]
- Use multi-scale OPE to disentangle perturbative corrections and reduce error
- In leading power of Δ/m_b a complete resummation of NNLO logarithms has been achieved[Neubert '04, Bosch et al. '04]
- NLO SM prediction reduced $\mathcal{O}(-5\%)$ [Neubert]

$$\mathrm{BR}_\gamma^{E_\gamma > 1.8\rm GeV} = (3.44 \pm 0.30) \times 10^{-4}$$

- New Calculation of spectrum [Melnikov '05]

NLO Complete

Need Matrix Elements at μ_b:

- The 1-loop ME Q_{1-6} give only a constant contribution: Absorb
- The LO result is then given by $C_7(\mu_b)$

At NLO:

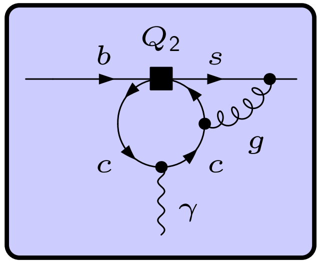
- 1-loop ME of $Q_{7,8}$
- 2-loop ME of Q_{1-6} [Greub, Hurth, Wyler '96, Buras et al. '01, Asatrian et al. '04]
- Bremsstrahlung [Ali, Greub '93; Pott '96]

Matching and Mixing and QED

- 2-loop matching
- 3-loop mixing [Chetyrkin, Misiak, Munz '97, Gambino, MG, Haisch '03]
- Look at QED corrections:
  - Use $\alpha_{em}^{onshell}$ [Czarnecki, Marciano '98]
  - Resum QED logs [Kagan, Neubert '99; Baranowski Misiak '00]
  - Matching reduces $\Gamma[b \to s\gamma]$ [Gambino, Haisch '00 '01]

The Charm Quark Issue

charm mass dependence starts at NLO

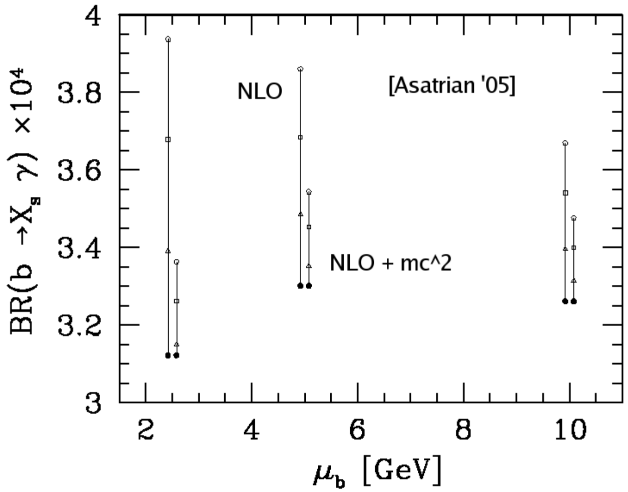


First charm dependent Matrix element is 2 loop:

- Formally, any definition for m_c can be used
- Gambino Misiak pointed out to use m_c in $\overline{\rm MS}$ at $\mu \sim m_b/2$: $\delta {\rm BR}_\gamma = +10\%$
- The dominant perturbative error is due to the charm quark mass

Have to do to NNLO

- To check the error reduction calculated the charm mass corrections [Asatrian '05]

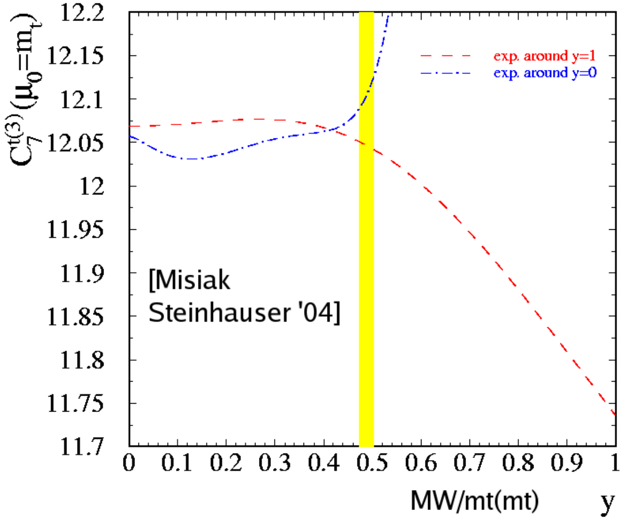


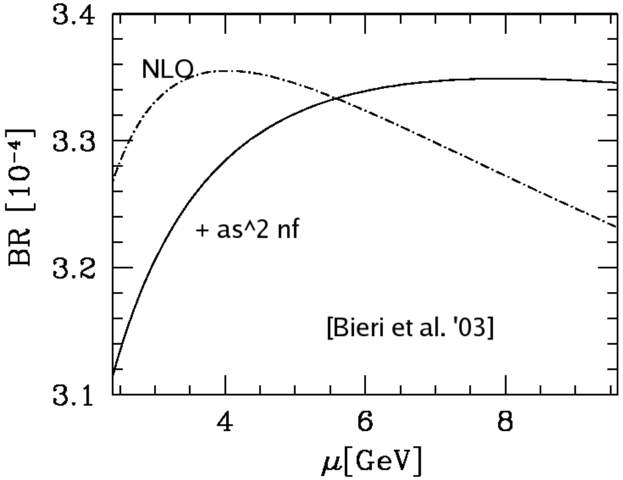
- Probably easier to use the RGE invariance

$$\delta A = \frac{\alpha_s}{4\pi} m_c \frac{d\ A}{d\ m_c} \gamma_m^{(0)} \ln \frac{\mu_b}{m_c}$$

NNLO prediction for $b \to s\gamma$ underway

- 2-loop matching of Q_{1-6} [Bobeth et al. '00]
- 3-loop matching of $Q_{7,8}$ [Misiak, Steinhauser '04]
- 3-loop mixing of Q_{1-6} [MG, Haisch '04]
- 3-loop mixing of $Q_{7,8}$ [MG, Misiak, Haisch '05]
- 4-loop mixing of Q_{1-6} into $Q_{7,8}$ [Czakon et al.]
- 2-loop matrix elements for $Q_{7,8}$ [Bieri et al. '03; Czarnecki et al.]
- 3-loop matrix elements of $Q_{1,2}$ [Bieri et al. Steinhauser '03; Misiak, Steinhauser]

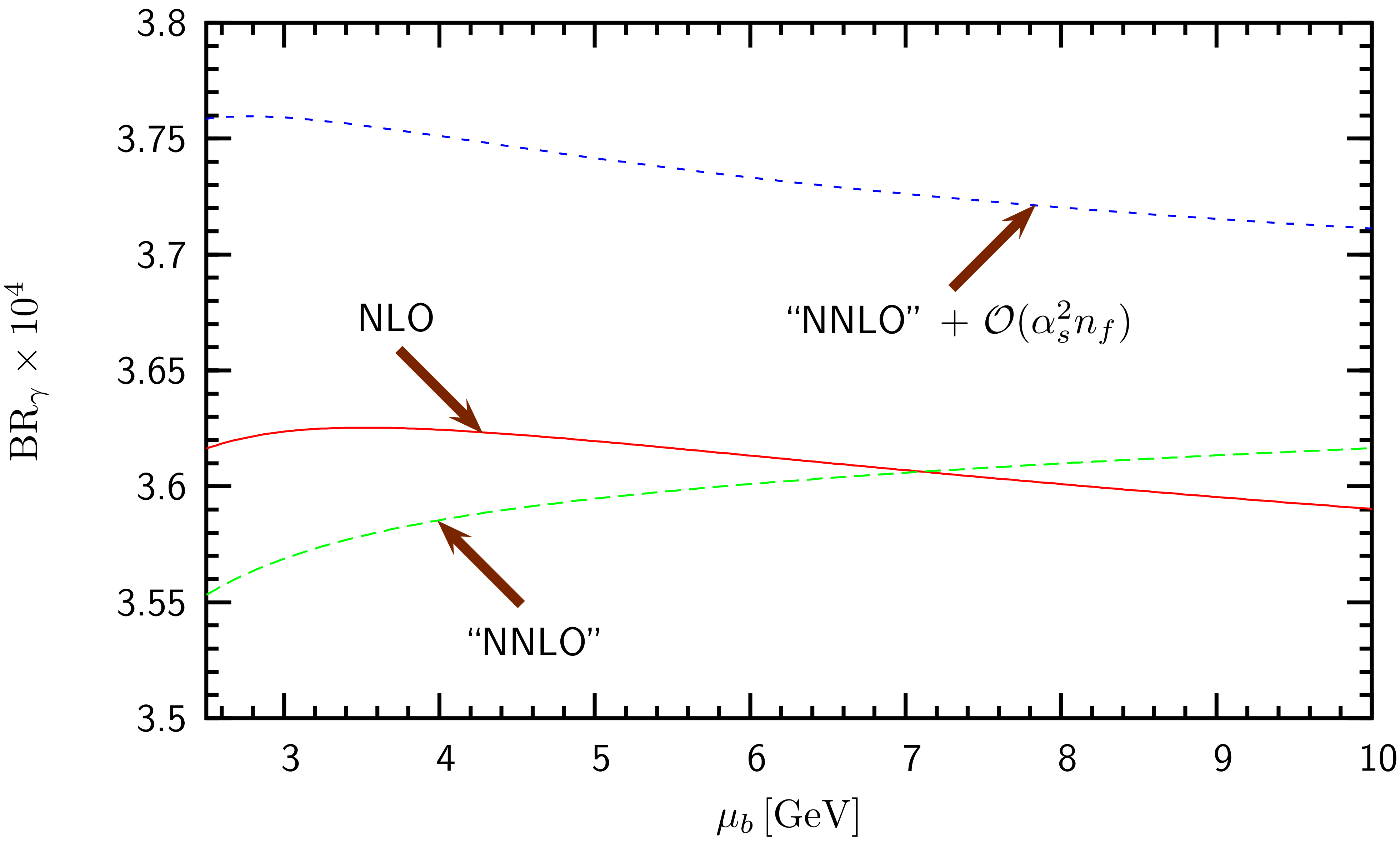




Results for the Anomalous Dimension Matrix

- Results were cross-checked by
  - Locality of UV divergences
  - By vanishing of mixing of non-physical into physical operators
  - completeness of the basis of effective operators
  - gauge-parameter independence
- We agree with Chetyrkin et al. for three-loop mixing of Q_1–Q_6 into Q_7 and Q_8 [Gambino et al. '03]
- We calculated for the first time the complete three loop mixing of Q_1–Q_10 [Gambino et al. '03; MG, Haisch '04; MG et al. '05] which represents an important step towards the completion of $b \longrightarrow s\gamma$
- We Calculated for the first time the QED corrections [Bobeth et al. '03] to the
  - one-loop and two-loop mixing of Q_9 and Q_{10}
  - two-loop mixing of Q_3^Q–Q_6^Q into Q_7–Q_{10}
  - two-loop mixing of Q_1–Q_6 into Q_9 and Q_{10}

NNLO still Incomplete



Error Anatomy of $\mathrm{BR}(\bar{B} \to X_s \gamma)$

- Following the analysis of Gambino Misiak

$$\begin{aligned}
\mathrm{BR}(\bar{B} \to X_s \gamma)_{E_\gamma > 1.6\mathrm{GeV}} = 3.61 \times 10^{-4} \times \\
(1 \pm 0.06_{m_c/m_b} \pm 0.04_{\mathrm{otherNNLO}} \\
\pm 0.01_{(\mathrm{pert\,C})} \pm 0.02_{\lambda_1} \pm 0.02_{\Delta} \\
\pm 0.02_{\alpha_s(M_Z)} \pm 0.02_{\mathrm{BR(semilept)_{exp}}} \pm 0.01_{m_t}) \\
= (3.61 \pm 0.30) \times 10^{-4}
\end{aligned}$$

- Total 8% error dominated by charm mass
- This will be improved by going to NNLO

The $\bar{B} \to X_s l^+ l^-$ decay

- Measured by BaBar and Bell: $\bar{B} \to X_s l^+ l^-$

$$\mathrm{BR}_{BaBar} = 5.6 \pm 1.5 \pm 0.6 \pm 1.1 \times 10^{-6} \qquad \mathrm{BR}_{Belle} = 4.11 \pm 0.83^{+0.85}_{-0.81} \times 10^{-6}$$

- Non-perturbative corrections can be controlled by
  - the heavy quark expansion for $\Lambda_{\mathrm{QCD}}/m_b$
  - kinematic cuts to avoid $c\bar{c}$ intermediate states $(B \to X_s \bar{c}c \to X_s l^+ l^-)$:

$$\text{low}: q^2 \equiv m^2_{l^+l^-} \in [1\mathrm{GeV}^2, 6\mathrm{GeV}^2]; \qquad \text{high}: q^2 > 14.4\mathrm{GeV}^2; \qquad \text{use}: \hat{s} = q^2/m_b^2$$

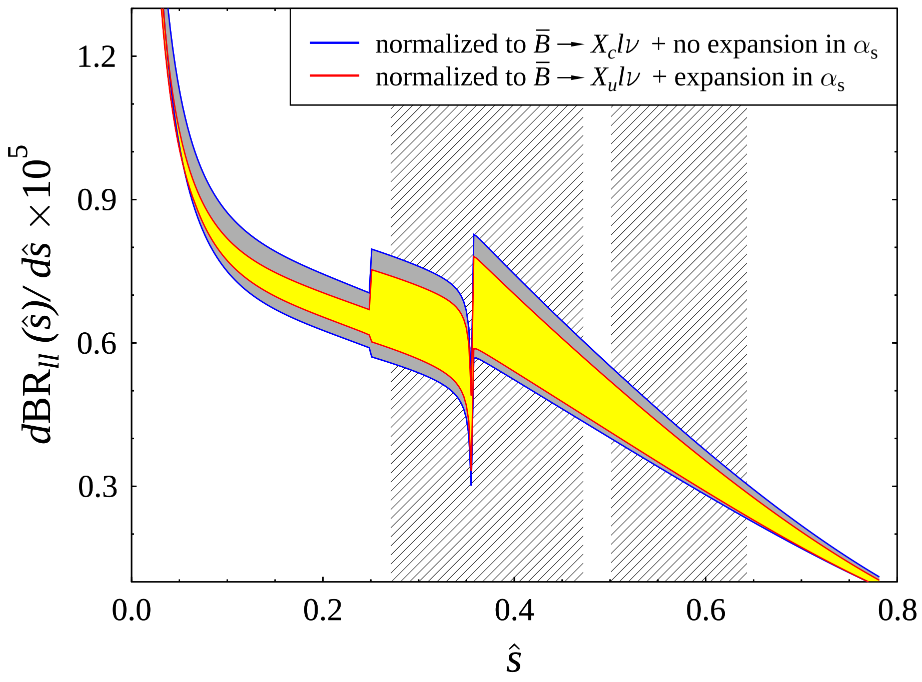
- To cancel m_b^5 dependence and avoid charm mass dependence normalise

$$\mathrm{BR}_{ll} = \frac{\mathrm{BR}[\bar{B} \to X_u l \bar{\nu}]}{C} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \int_{0.05}^{0.25} d\hat{s} \frac{d\Gamma[\bar{B} \to X_s l^+ l^-]/d\hat{s}}{\Gamma[\bar{B} \to X_u l \bar{\nu}]}$$

Completing the NNLO Analysis of $\bar{B} \to X_s l^+ l^-$

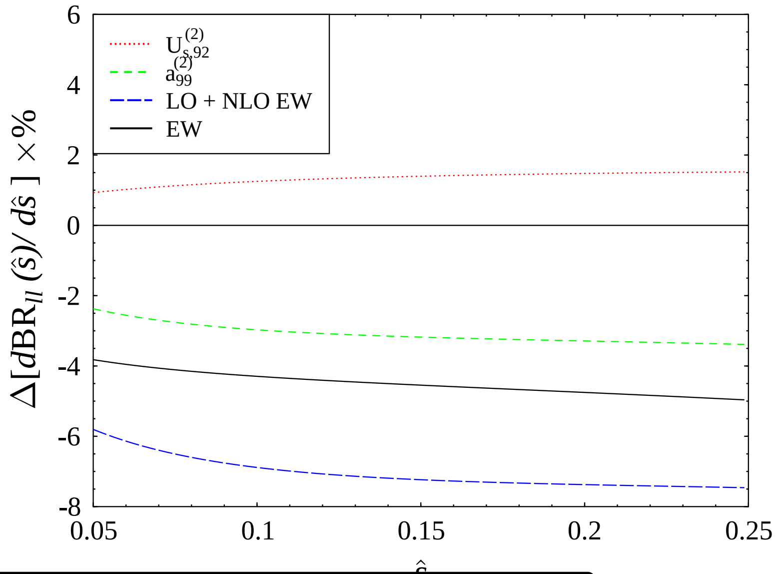
Recently the NNLO Calculation has been (nearly) completed

- 2-loop matching conditions [Bobeth, Misiak, Urban '00]
- 2-loop matrix elements of Q_1,Q_2 and bremsstrahlung [Asatrian et al. '02 '03; Ghinculov et al. '03]
- 2-loop matrix element of Q_9 [Bobeth, Gambino, Gorbahn, Haisch '03]
- 3-loop evolution [Gambino, Gorbahn, Haisch '03]
- 2-loop matrix elements of Q_1 and Q_2 for the high q^2 region [Ghinculov, Hurth, Isidori, Yao '03]



Further Improvements for $\bar{B} \to X_s l^+ l^-$

- Normalising to $b \to X_u e \bar{\nu}_e$ and expansion in α_s reduces theoretical uncertainty by 50%
- As a result of accidental cancellations, new NNLO QCD and EW corrections change the low q^2 BR by -3% and -2%
- The Inclusion of EW effects reduces 8% uncertainty due to previously unknown EW effects [Bobeth, Gambino, Gorbahn, Haisch '03]



In the low q^2 region parametric uncertainties dominate the error

Comparing Theory and Experiment

- Integrating over low-q^2 region:

$$\mathrm{BR}_{1<q^2/\mathrm{GeV}^2<6} = (1.57 \pm 0.11|_{m_t} \pm 0.07|_{m_b} \pm 0.07|_{\mu} \pm 0.05|_{c\bar{c}} \pm 0.05|_{C}) \times 10^{-6}$$

- Integrating over high-q^2 region:

$$\mathrm{BR}_{14.4<q^2/\mathrm{GeV}^2} = (4.02 \pm 0.71|_{m_b} \pm 0.24|_{m_t} \pm 0.13|_{\mu} \pm 0.13|_{c\bar{c}} \pm 0.12|_{C}) \times 10^{-7}$$

- Integrating the non-resonant differential rate

$$\mathrm{BR}_{SM} = (4.58 \pm 0.18 \pm 0.66) \times 10^{-6}$$

- Agrees with experiments

$$\mathrm{BR}_{BaBar} = 5.6 \pm 1.5 \pm 0.6 \pm 1.1 \times 10^{-6} \qquad \mathrm{BR}_{Belle} = 4.11 \pm 0.83^{+0.85}_{-0.81} \times 10^{-6}$$

$$\mathrm{BR}^{<6\mathrm{GeV}}_{BaBar} = 1.8 \pm 0.7 \pm 0.5 \times 10^{-6} \qquad \mathrm{BR}^{<6\mathrm{GeV}}_{Belle} = 1.493 \pm 0.503^{+0.382}_{-0.283} \times 10^{-6}$$

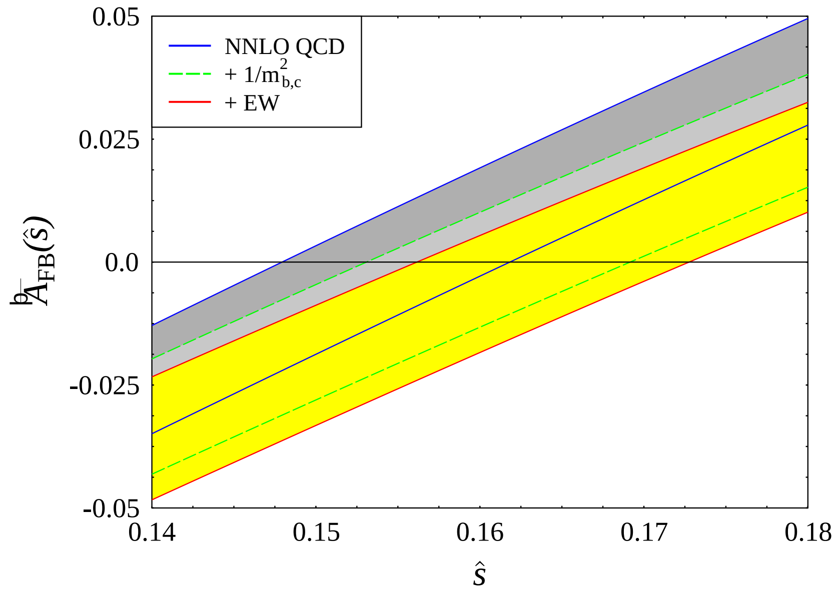
Forward-Backward Asymmetry

$$\bar{A}_{\rm FB}(\hat{s}) = \frac{1}{d\Gamma[\bar{B} \to X_s l^+ l^-]/d\hat{s}} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\Gamma[\bar{B} \to X_s l^+ l^-]}{d\hat{s}\, d\cos\theta_\ell} \, \mathrm{sgn}(\cos\theta_\ell)$$

- Position of FB asymmetry affected by sign and magnitude of C_7/C_9

$$q^2_{0,SM} = (3.76 \pm 0.22 \pm 0.24)\,\mathrm{GeV}^2$$

- NNLO QCD and EW corrections enhance result by 15% and reduce uncertainties from $\pm 20\%$ to $\pm 6\%$
- Zero of the FB asymmetry provides sensitive test of new physics



Conclusions

$\bar{B} \to X_s l^+ l^-$

- NNLO calculation of $\bar{B} \to X_s l^+ l^-$ is completed
- Uncertainty due to 8% EW effects resolved
- The extrapolated $\mathrm{BR}_{SM} = (4.58 \pm 0.18 \pm 0.66) \times 10^{-6}$ agrees with the experiment
- FB asymmetry provides sensitive test of NP

$\bar{B} \to X_s \gamma$

- The Standard Model is consistent with the current experimental data
- The main uncertainty of the theory resides in the perturbative side (m_c)
- NNLO calculation will solve this