# Theory Status the rare K decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

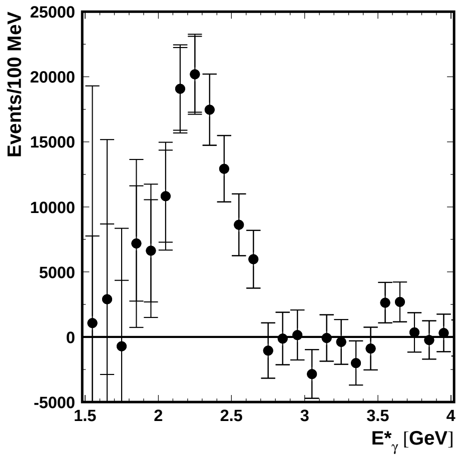
Martin Gorbahn
IPPP University of Durham

- Error of $b \to s\gamma$ decays
- Why look at rare K decays
- Status of $K^+ \to \pi^+\nu\bar{\nu}$
- $K^+ \to \pi^+\nu\bar{\nu}$ at NNLO
- Adding $K_L \to \pi^0\nu\bar{\nu}$
- The Future

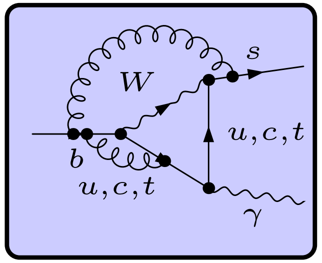
**WIN 2005**

# Theory error in $b \to s\gamma$

- We can only observe decays of bound states $\Rightarrow$
- Study inclusive decay




- Theory or Experimental error is model dependent
- Still provides precision test of $b \to s$ transitions




For n gluons we have

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^n \frac{m_b^2}{M_W^2} (LL)$$

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^{n-1} \frac{m_b^2}{M_W^2} (NLL)$$

- Large logs $\Rightarrow$ straightforward perturbation theory unreliable
- Leading Log enhance Branching Ratio by $200\%$
- Use renormalisation group to resum leading and next-to-leading logs
- Will be even better by going to NNLO

# The decay $K^+ \to \pi^+ \nu \bar{\nu}$

- Short distance dominated $\to$ Theoretically clean

- Present NLO calculation

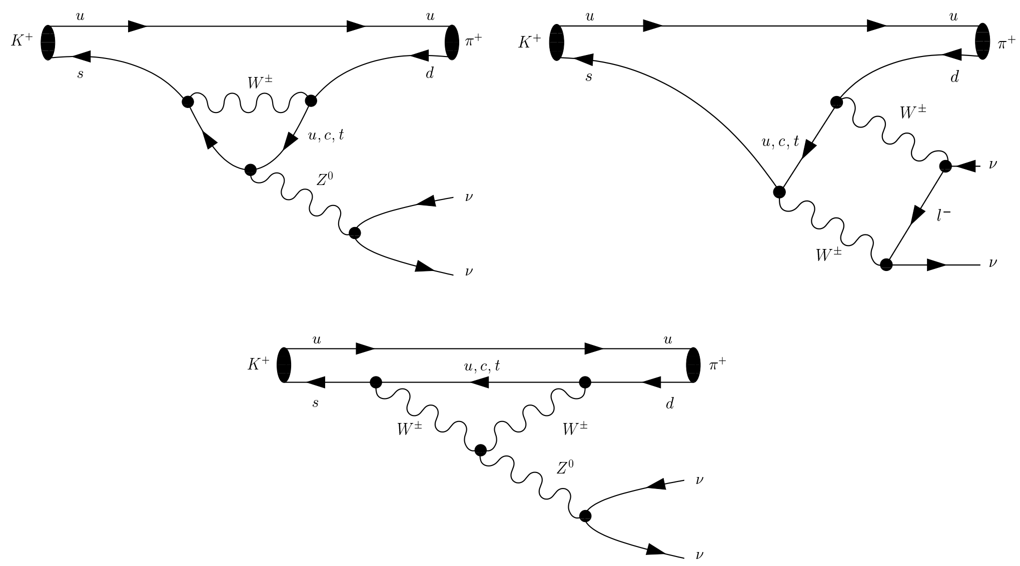
$$BR^{NLO}_{K^+\nu\bar{\nu}} = (7.8 \pm 1.2) \times 10^{-11}$$

  Main uncertainty resides in the charm mass scheme dependence $\to$ can be improved by a NNLO calculation

- First measurement by AGS E787 '02 and new one by AGS E949 '04

$$BR^{NLO}_{K^+\nu\bar{\nu}} = (14.7^{+13.0}_{-8.9}) \times 10^{-11}$$

- This error should improve in the future




- If theory and experimental error reduce this decay will provide a precision test of the $s \to d$ transitions

# Basic Structure

- Calculation of the penguin and box diagrams

$$\sum_i V_{is}^* V_{id} F(x_i) \; = \; V_{cs}^* V_{cd} (F(x_c) - F(x_u)) + V_{ts}^* V_{td} (F(x_t) - F(x_u))$$

$$\sim \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c} \qquad\qquad \sim \frac{m_t^2}{M_W^2} \times \text{CKM suppressed}$$

- The effective Hamiltonian is ($\lambda$ is the Cabibbo angle):

$$\boxed{H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{2}{2\pi \sin^2 \Theta_W} (V_{cs}^* V_{cd} X_c(X) + V_{ts}^* V_{td} X(x_t))(\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}}$$

- Matrix Element can be related to the semileptonic decay $K^+ \to \pi^0 e^+ \nu$
  - the isospin breaking effects have been calculated $\to r_{K^+} = 0.901 \quad r_{K_L} = 0.944$

    [Marciano, Parsa '96]

## The $K \to \pi\nu\bar{\nu}$ Branching Ratio

- The branching ratio is then given by $(\lambda_i = V^*_{is} V_{id})$:

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \left( \left( \tfrac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 + \left( \tfrac{\mathrm{Re}\lambda_c}{\lambda} P_c(X) + \tfrac{\mathrm{Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 \right)$$

$$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) = \kappa_L \left( \tfrac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t) \right)^2$$

- Where $\kappa$ is precisely known:

$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathrm{BR}(K^+ \to \pi^0 e^+ \nu)}{2\pi^2 \sin^2\theta_W} = (4.84 \pm 0.06) 10^{-11} \times \left( \frac{\lambda}{0.224} \right)^8$$

$$\kappa_L = \kappa_+ \frac{r_{K_L} \tau(K_L)}{r_{K^+} \tau(K^+)} = (2.12 \pm 0.03) 10^{-11} \times \left( \frac{\lambda}{0.224} \right)^8$$

# Theory Uncertainties of $K^+ \to \pi^+ \nu\bar{\nu}$

| Perturbative | Non-Perturbative |
| --- | --- |

- $X(x_t)$: The top contribution
  - No large logs $\to$ ordinary perturbation theory [Buchalla, Buras '93]
  - Scale uncertainty at two loop of $\pm 1\%$
- $P_c(X)$: The charm contribution
  - Contains a large log $\ln \frac{m_c}{M_W}$
  - $Z^0$-penguin and box contribution
  - Large Logs are resummed up to NLO [Buchalla, Buras '93]

$$P_c(X) = 0.389 \pm 0.033_{m_c} \pm 0.045_{\mu_c} \pm 0.01_{\alpha_s}$$

- Soft u quarks in the penguin loop
  - use chiral perturbation theory to calculate contributions to the $K^+ \to \pi^+ Z$ vertex
  - Sum of octet part of $\mathcal{O}(5\%)$ vanishes [Ecker et al. '88]
  - In the large $N_c$ limit total contribution cancels [Lu, Wise '94]
- Contributions of higher dimensioal Ops
  - Calculated using CHPT [Isidori et al. '05]
  - $P_c(X) \to P_c(X) + 0.04 \pm 0.02$

# Theoretical Framework: Effective Field Theories

At high scales $\mu_0 \sim M_W$ the full theory contains heavy $W, t, \ldots$ and light $g, b, \ldots$ fields:

$$\mathcal{L}_{\rm full} = \mathcal{L}_H(h, l) + \mathcal{L}(l).$$

At a low scale $\mu < \mu_0$ we obtain an effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \mathcal{L}(l) + \delta\mathcal{L}(l)$$

The calculation takes three steps

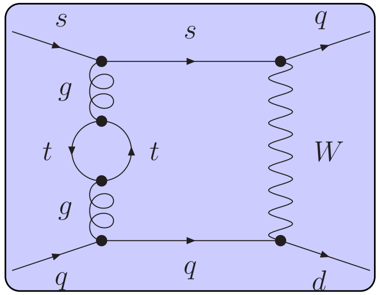
- Matching of $\mathcal{L}_{\rm full}$ and $\mathcal{L}_{\rm eff}$ at $\mu_0$ gives $\delta\mathcal{L}(L)$
- With the help of the Renormalisation Group Equation (RGE) we can relate the effective Lagrangian at the high scale to the low scale one

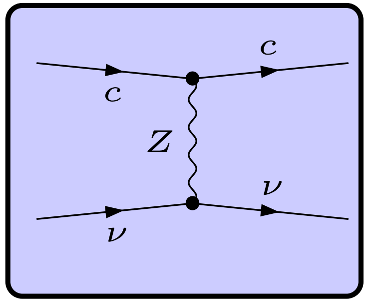
$$\mathcal{L}_{\rm eff} \quad \text{at} \quad \mu_0 \rightarrow \mathcal{L}_{\rm eff} \quad \text{at} \quad \mu$$

- Calculation of the matrix elements (process dependent)

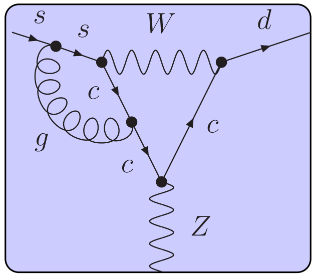
# Penguin Sector at NNLO

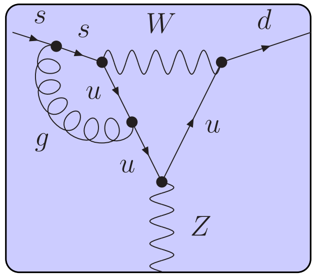
- Two loop Matching of $|\Delta S| = 1$ interactions and the Z boson will generate the Wilson Coefficients of three dimension 6 operators: $Q_1$, $Q_2$, $Q_Z$ [Bobeth et al '00; Buras et al.]



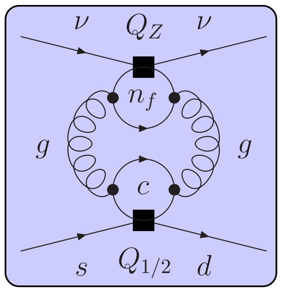



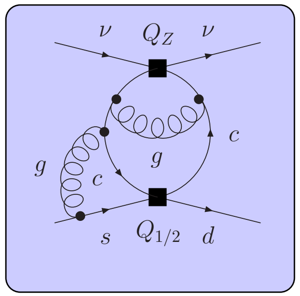

- Two loop Matching of the Z penguin will generate the $K \to \pi\nu\bar{\nu}$ operator $Q_K = m_c^2(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ [Buras et al.]




- 3 loop mixing of the dimension 6 operators via bi-local insertions of $\langle s\nu|Q_{1/2}Q_Z|d\nu\rangle$ into $Q_K$ [Buras et al.]



# Calculation of the Anomalous Dimension Matrix

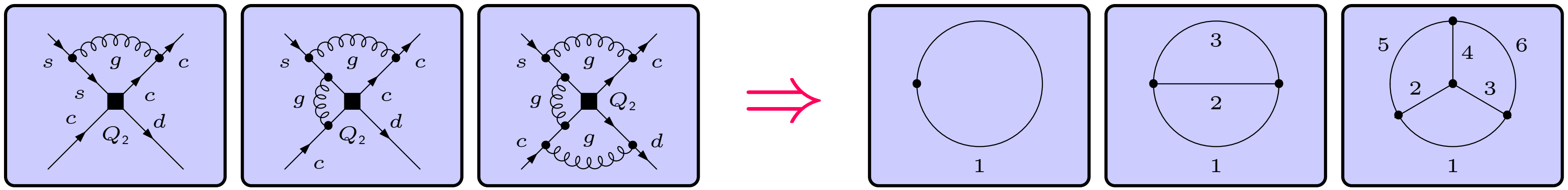
- We are interested in the UV divergences, which are polynomial in the external Momenta

The divergences are polynomial → Exact decomposition of the propagator

Expansion ⇒ spurious IR divergences → Exact decomposition of the propagator

$$\frac{1}{(k+p)^2 - m^2} = \frac{1}{k^2 - M^2} - \frac{p^2 + 2k\cdot p - m^2 + M^2}{k^2 - M^2}\,\frac{1}{(k+p)^2 - m^2}\,.$$

- By repeating this decomposition the divergent part of the integral is rewritten as a tensor vacuum integrals with one common mass



## Complete NNLO analysis

- Calculation in the box sector is also finished
- Included threshold corrections at $m_b$
- Still have to do Numerics
- Stay tuned and look out for [Buras, MG, Haisch, Nierste]
- Result will (hopefully) reduce scheme dependence drastically
- But what shall we do with it?

# The decay $K_L \to \pi^0 \nu \bar{\nu}$

Perturbatice

Non-Perturbative

- Leading decay proceeds through CP violation
  - $\propto \mathrm{Im}\lambda_t$
  - No GIM suppression of $X(x_t)$ compared to charm sector
  - Intrinsic theory error much smaller
- Effective Hamiltonian same as for $K^+ \to \pi^+ \nu \bar{\nu}$
  - Again using Isospin relate to $K^+ \to \pi^0 e^+ \nu$

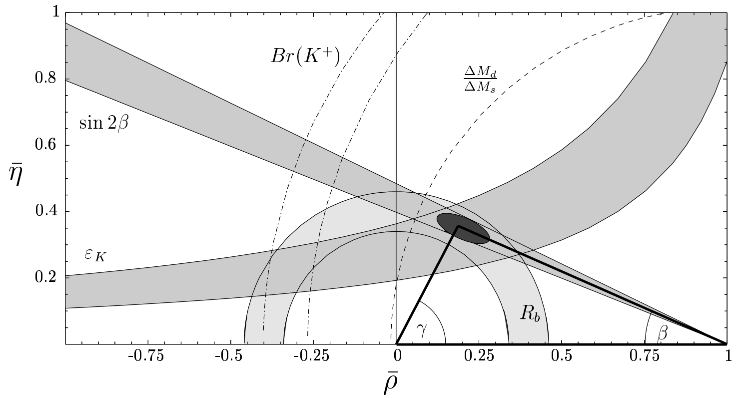
$$BR(K_L)_{\mathrm{SM}} = (3.0 \pm 0.6) 10^{-11}$$

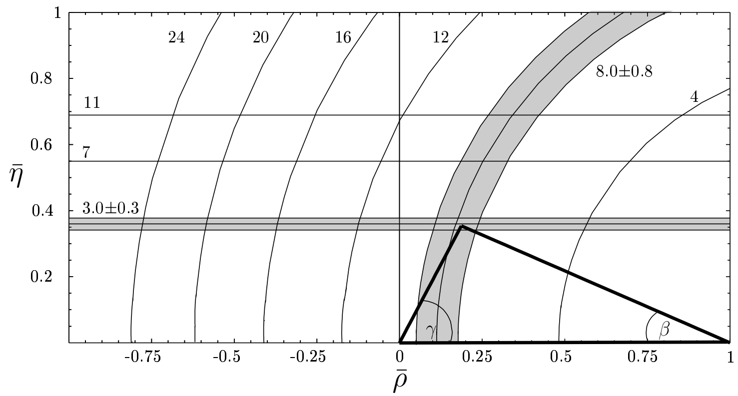
There are CP conserving contributions even in the SM

- From soft u quarks
  - Contribution supressed by $10^{-5}$ [Buchalla, Isidori]
- From higher dimensional Operators
  - Leading contribution vanishes in CHPT
  - estimated to be $10^{-5}$ effect [Buchalla, Isidori]

# Future scenario for $K \to \pi\nu\nu$

- $K_L \to \pi^0 \nu\bar{\nu}$
  - Direct CP violating $\propto \mathrm{Im} V_{ts}^* V_{td} = \eta$
  - Only top $\to$ theory uncertainty $< 3\%$
- $K^+ \to \pi^+ \nu\bar{\nu}$
  - $\propto V_{ts}^* V_{td}$
  - Small theory uncertainty
- Experiment
  - Precision test of the Unitarity Triangle
  - And the Flavour Sector



# Even better

- Determination of the unitarity triangle independent of $V_{ub}$ and $V_{cb}$:
  - $\mathrm{Im}\lambda_t = \lambda^5 \sqrt{\mathrm{BR}_{\mathrm{K_L}}/\kappa_{\mathrm{L}}}/X(x_t)$
  - $\mathrm{Re}\lambda_t = -\lambda^5 (P_c(X)\mathrm{Re}\lambda_c/\lambda + \sqrt{\mathrm{BR}_{\mathrm{K^+}}/\kappa_+ - \mathrm{BR}_{\mathrm{K_L}}/\kappa_{\mathrm{L}}})/X(x_t)$
  - Dominant error in $P_c$
- will be significantly reduced by NNLO calculation
- Similar for the $\sin 2\beta$ determination

# Conclusions

- $K^+ \to \pi^+ \bar{\nu}\nu$
  - Contribution of higher dimensional Operators calculated
  - NNLO calculation completed
  - Intrinsic theory error small
- $K_L \to \pi^0 \bar{\nu}\nu$
  - Theoretically very clean
  - Provides together with $K^+ \to \pi^+ \bar{\nu}\nu$ a great test of the flavour sector
- Looking forward to the Experiments