Theory overview of B Mixing and Lifetimes

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OUTLINE

Theoretical inputs for

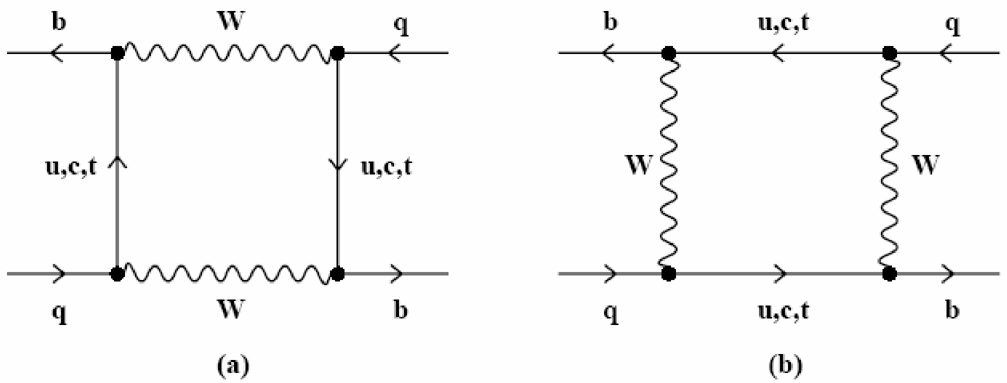
- Mixing parameters: Δm_d, Δm_s
- Width Differences: $\Delta\Gamma_d$, $\Delta\Gamma_s$
- Lifetime Ratios: $\tau(B^+)/\tau(B_d)$, $\tau(\Lambda_b)/\tau(B_d)$

Many thanks for the invitation

WIN 05, Delphi, Greece June 6 - 11

Basic Formalism

Neutral mesons are not eigenstates of the Weak Interactions:



$\Rightarrow$ "particle-antiparticle oscillations":

- Schroedinger equation with an effective 2×2 Hamiltonian

$$i\frac{d}{dt}\begin{pmatrix} B_q \\ \overline{B}_q \end{pmatrix} = \left[\begin{pmatrix} M_{11}^q & M_{21}^{q\,*} \\ M_{21}^q & M_{11}^q \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11}^q & \Gamma_{21}^{q\,*} \\ \Gamma_{21}^q & \Gamma_{11}^q \end{pmatrix}\right]\begin{pmatrix} B_q \\ \overline{B}_q \end{pmatrix}$$

- Mass eigenstates:

$$|B_q^{L,H}\rangle = (|B_q\rangle \pm (q/p)_q|\overline{B}_q\rangle)/\sqrt{1+|(q/p)_q|^2}.$$

Physical Observables

$$\Delta m_q = 2\,|M_{21}^q|$$

$$\Delta\Gamma_q = -2\,|M_{21}^q|\,\mathrm{Re}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

$$\Gamma_q = \tau_q^{-1} = |\Gamma_{11}^q|$$

$$\left|(q/p)_q\right| = 1 + \frac{1}{2}\,\mathrm{Im}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

1. Mixing

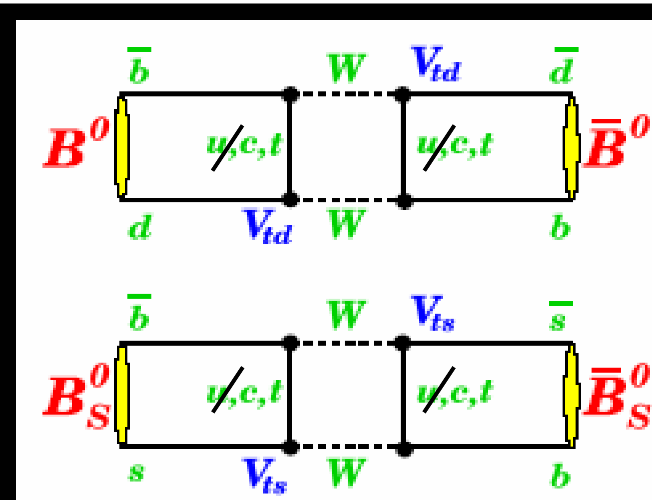
Δm_d, $\Delta m_s / \Delta m_d$

- Decay Constants and Bag Parameters
- SM phenomenology (UTA)

Extrating $|V_{td}|$:

$$\Delta m_d = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_d} \left|V_{td}\right|^2 \eta_B S_0(x_t) \cdot \hat{B}_{B_d} f_{B_d}^2$$

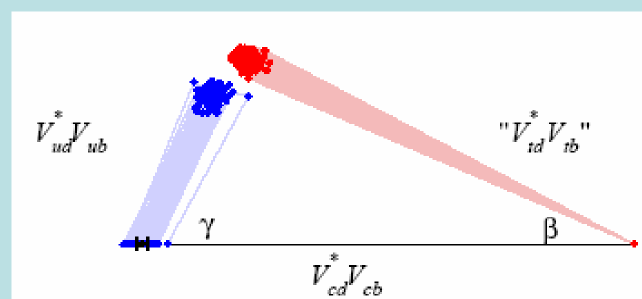
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \left|\frac{V_{ts}}{V_{td}}\right|^2 \left(\frac{\hat{B}_{B_s} f_{B_s}^2}{\hat{B}_{B_d} f_{B_d}^2}\right) \leftarrow \xi^2$$



- Top contribution is dominant: OPE $(m_t \to \infty, m_W \to \infty)$
- $\eta_B S_0(x_t)$: short distance physics - computable in P.T (2%)

  @NLO Buras et al: Nucl. Phys. B347 (1990)
- $\hat{B}_B f_B^2$: Non-Perturbative QCD effects - Lattice QCD and QCD Sum Rules
- $\Delta m_s / \Delta m_d$: short distance effects and lattice uncertainties cancel

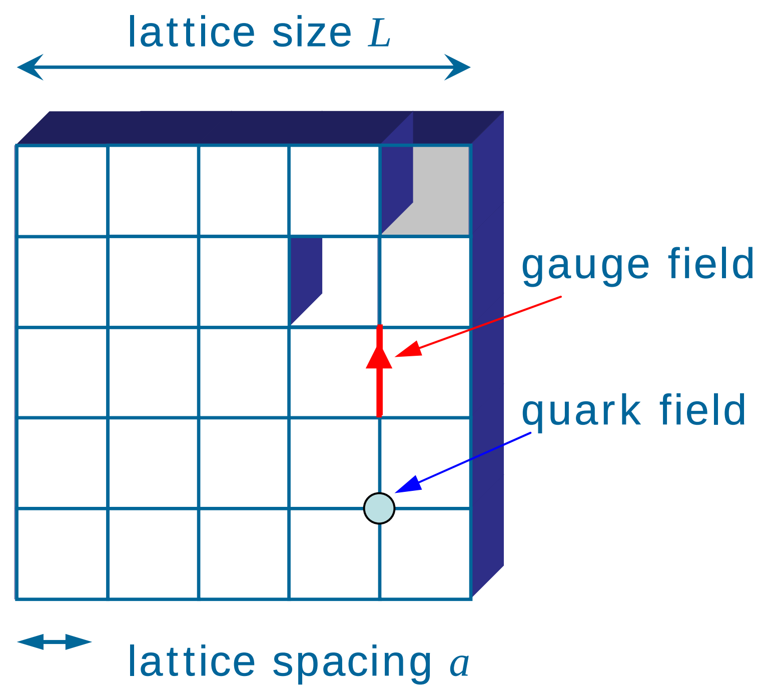
Lattice QCD is the closest tool to achieve model-independent calculations of B_B *and* f_B



Lattice QCD

- Lattice QCD is Quantum Field Theory on a Finite and Discrete Box;

$$Z_{QCD} = \int^{V} \mathcal{D}A_\mu e^{-S_g} \prod_q \det(\not{D} + m_q)$$



• Physical Quantities are computable from first principles, *by tuning only the parameters appearing in the QCD Lagrangian, namely* m_q *and* α_s

• However, although no ``ab initio'' limitations on the approach, limitations in computing resources introduce some approximations:

source of systematic errors.

Systematic Errors I

IR – cut-off

$$\frac{1}{L} \ll m_q \ll \frac{1}{a}$$

UV – cut-off

(discretization errors)

$1/L \sim 100 - 200\,\mathrm{MeV}$

m_s but m_{ud}

$\left(\begin{array}{c} 1/2 \leq m_l / m_s \leq 2 \\ \text{accessible region} \end{array} \right)$

Current Simulations

$1/a \sim 2 - 4\,\mathrm{GeV}$

m_c but m_b

$\left(\begin{array}{c} m_c \leq m_H \ll m_b \\ \text{accessible region} \end{array} \right)$

To face with

Chiral Extrapolation??

Heavy Extrapolation??

help from ChPT and HQET

To get B-physics on the lattice, 2 main routes

First of all, to avoid large discretization errors *(am_H, $a\Lambda_{QCD}$)*

O(a) improved actions and operators

Working at several ``a'' and go to $a \to 0$

a) Relativistic Approach: [rather large errors]

- Compute **QCD** for the accessible heavy quarks,
- Extrapolate to $\mathbf{1/m_B}$ with heavy quark scaling law (APE,UKQCD)

recent improvement: combine data simulated from HQET ($m_b \to \infty$) (SPQcdR),

: Non-perturbatively renormalised in HQET devised (ALPHA)

b) Effective Theory Approach (NRQCD and FNAL): [unknown uncertainties]

- Some of the coefficients, in the action and the operators, are known only in free theory
- Difficulties with higher orders $1/(am_H)$: ``renormalon shadow'', cancellation of power divergences, ``no continuum limit''

FINAL ERROR: COMPARE RESULTS OR COMBINE DIFFERENT APPROACHES

Light Quarks on the lattice: technical issue

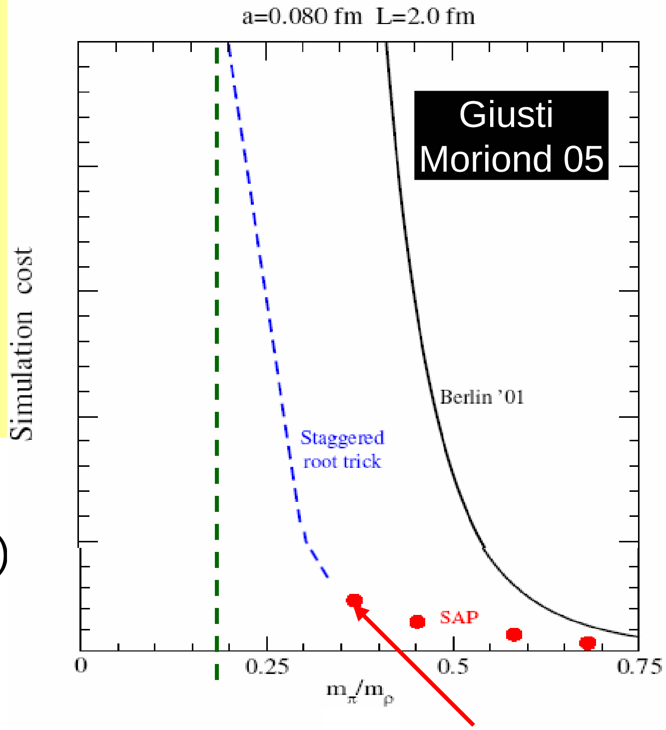
Simulation costs for light masses are very demanding for $m_q/m_s<0.5$ (m_π< 500 MeV)

For the Wilson formulation of the QCD action (the most used), we have: $\mathrm{Cost} \propto N_{\mathrm{conf}}\, m_q^{-3}\, L^5\, a^{-8}$

Recently:

Staggered ☆ Action can push m_q/m_s to 0.25 (*FV potentially large!!*)

TmQCD and SAP-Wilson can reach this goal as well *(exploratory studies so far)* $\mathrm{Cost}^{\mathrm{SAP}} \propto N_{\mathrm{conf}}\, m_q^{-1}\, L^5\, a^{-6}$



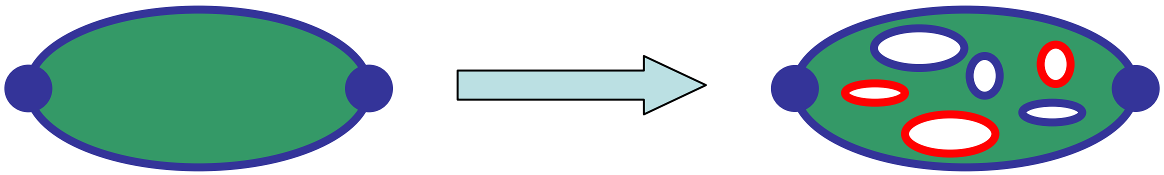
M_π=276 MeV

QCD Lat Actions	chiral symmetry	flavor symmetry	Comp. Cost	
Wilson/*O(a)*-improved Wilson	*violated*;recovered in the continuum	O.K.	*expensive*; harder at lighter masses	1973/1996
twisted mass	violated	2 flavors; a flavor mixing mass term	less expensive at lighter masses	2001
staggered ☆	exact U(1) out of U(4)	*4 tastes*; non-trivial mixing	*fast with the fourth root trick* ☆	2001
Domain-wall/ Overlap	exact at finite *a*	O.K	most expensive; still exploratory	1997

Systematic Errors II (Dynamical Fermions)

$$Z_{QCD} = \int \mathcal{D}A_\mu e^{-S_g} \prod \det(\not{D} + m_q)$$

Quenched: neglect it



QUENCHED QCD

QCD

- ``Unquenched'' results are becoming available;
- They are partially quenched $m_{val} \neq m_{sea}$;
- **Up to now,** simulations are made by using Wilson or Staggered actions for light quarks and NRQCD or FNAL for the heavy.

Current B-physics Run:

(1/a~2 GeV, 1/L~90 MeV)

JLQCD-MILC-CP-PACS:
$N_F=2$, $m_q/m_s > 0.5$, $m_\pi \sim 500$ MeV

HPQCD+MILC:
$N_F=2+1$, $m_q/m_s > 0.2$, $m_\pi \sim 300$ MeV

Systematic uncertainties improvable: brute force (TFlop computers) and algorithms

Bs-MESON DECAY CONSTANT on LATTICE QCD

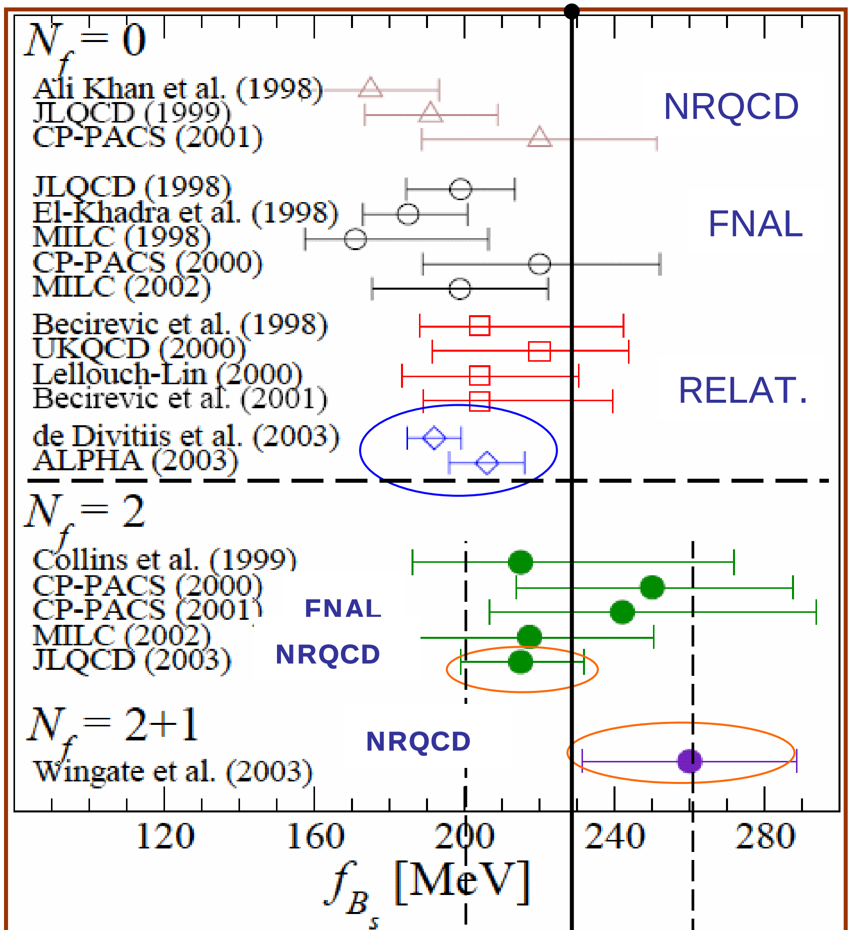
$$\langle 0|\bar{b}\gamma^{\mu}\gamma_5 s|B_s\rangle = ip^{\mu} f_{B_s}$$

Quenched Simulations

- ✓ High level of accuracy;
- ✓ Good agreement among different approaches;
- ✓ Continuum extrapolation; **(De Divitiis et al. (2003), ALPHA (2003))**

Unquenched Simulations

- **N_F=2:** some studies (JLQCD (03), high statistics and O(*a*)-improved action);
- **N_F=2+1:** preliminary values; Wingate et al (03), staggered action



With the present accuracy

- sea quark effects seem at *O*(10%-15%):
- But continuum scaling or other systematic (NP-ren., FV effects, diff. actions …) not completely investigated

ICHEP 2004:

$$f_{B_s} = 230 \pm 30 \text{ MeV}$$

Bd-MESON DECAY CONSTANT on LATTICE QCD

$$\langle 0 | \bar{b}\gamma^{\mu}\gamma_5 d | B_d \rangle = i p^{\mu} f_{B_d}$$

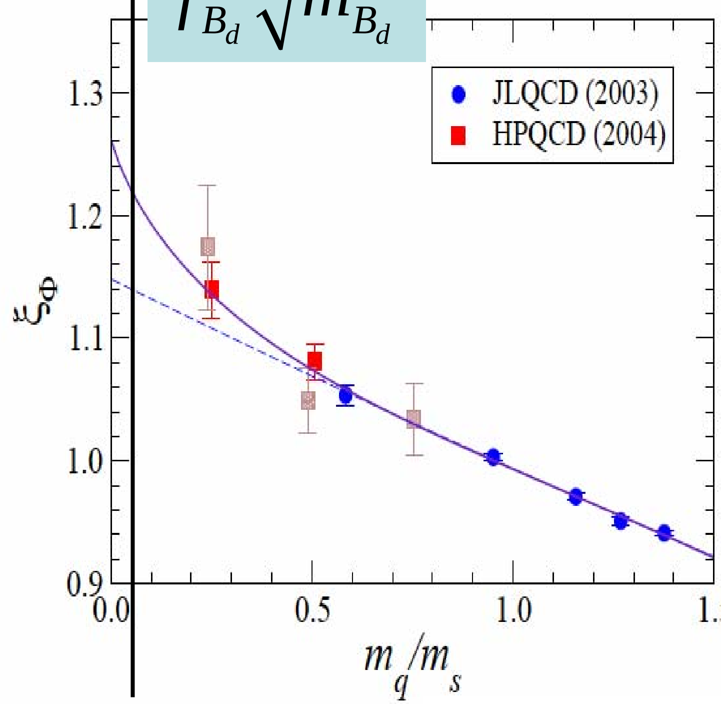
Delicate Issue: Chiral Extrapolation

- Pion Loops could be significant
  [Kronfeld-Ryan (02), *O*(20%)]

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}} \propto 1 + \frac{3(1+3g^2)}{4}\frac{m_\pi^2}{(4\pi f)^2}\log\frac{m_\pi^2}{\mu^2}$$

$$(1+3g^2) \sim 1.8$$

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}}$$



With the present accuracy:

- Chiral Log Effects roughly estimated
  (JLQCD (N_F=2, m_q/m_s>0.5)
  + HPQCD (N_F=2+1, m_q/m_s>0.2))

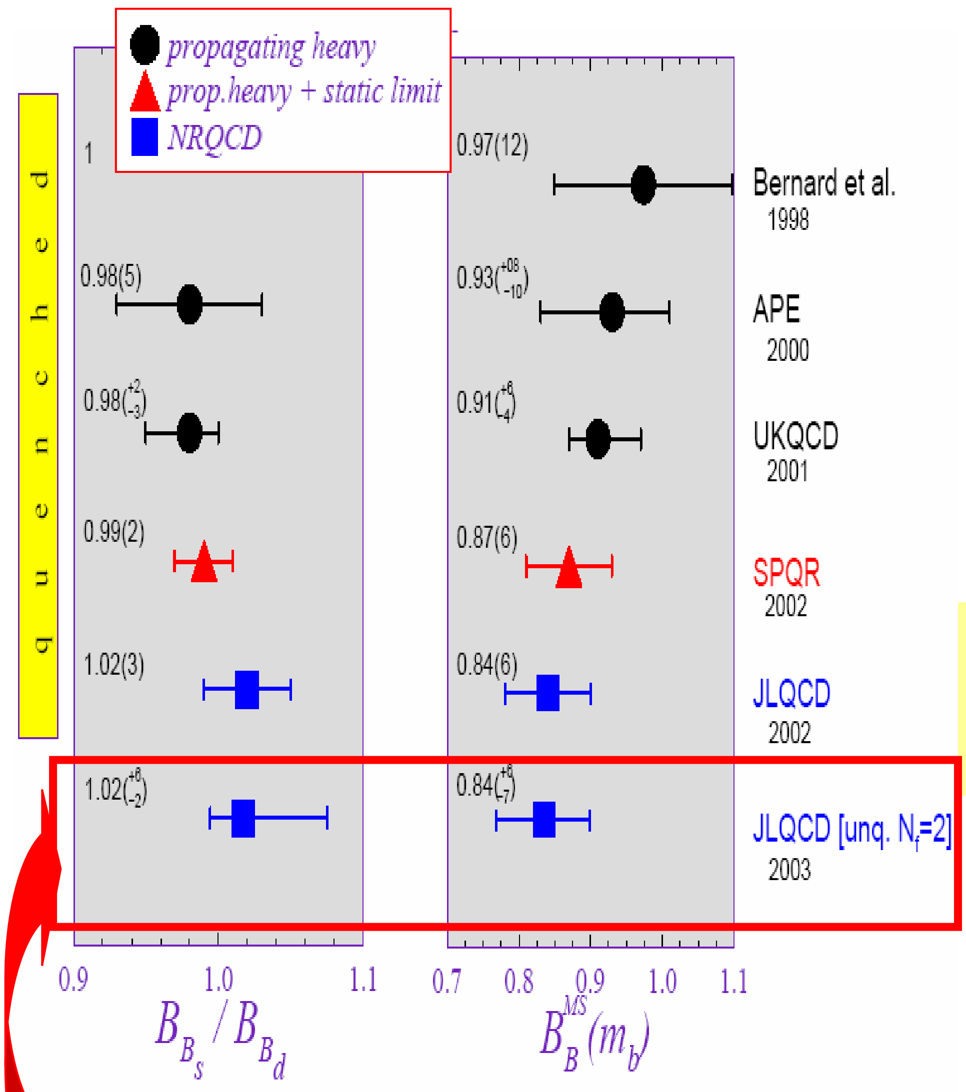
ICHEP 2004: $\frac{f_{B_s}}{f_B} = 1.22^{+0.05}_{-0.06}$

Still more statistics and lighter quark masses is needed

Final Chiral Loop Effects: *O*(10%)

B-$\bar{\text{B}}$ MIXING ON THE LATTICE

$$\langle \overline{B}_q | (\bar{b}\gamma_L^\mu q)(\bar{b}\gamma_L^\mu q) | B_q \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$



JLQCD(03): $N_F=2$ & NRQCD

With the present accuracy:

- No sea quark effects!!
- Consistent with diagnosis deduced from QChPT versus ChPT: Booth95,Sharpe,Zhang96

$$\hat{B}_{B_d} = 1.287(42)\left({}^{+86}_{-95}\right), \quad \frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.017(16)\left({}^{+56}_{-17}\right)$$

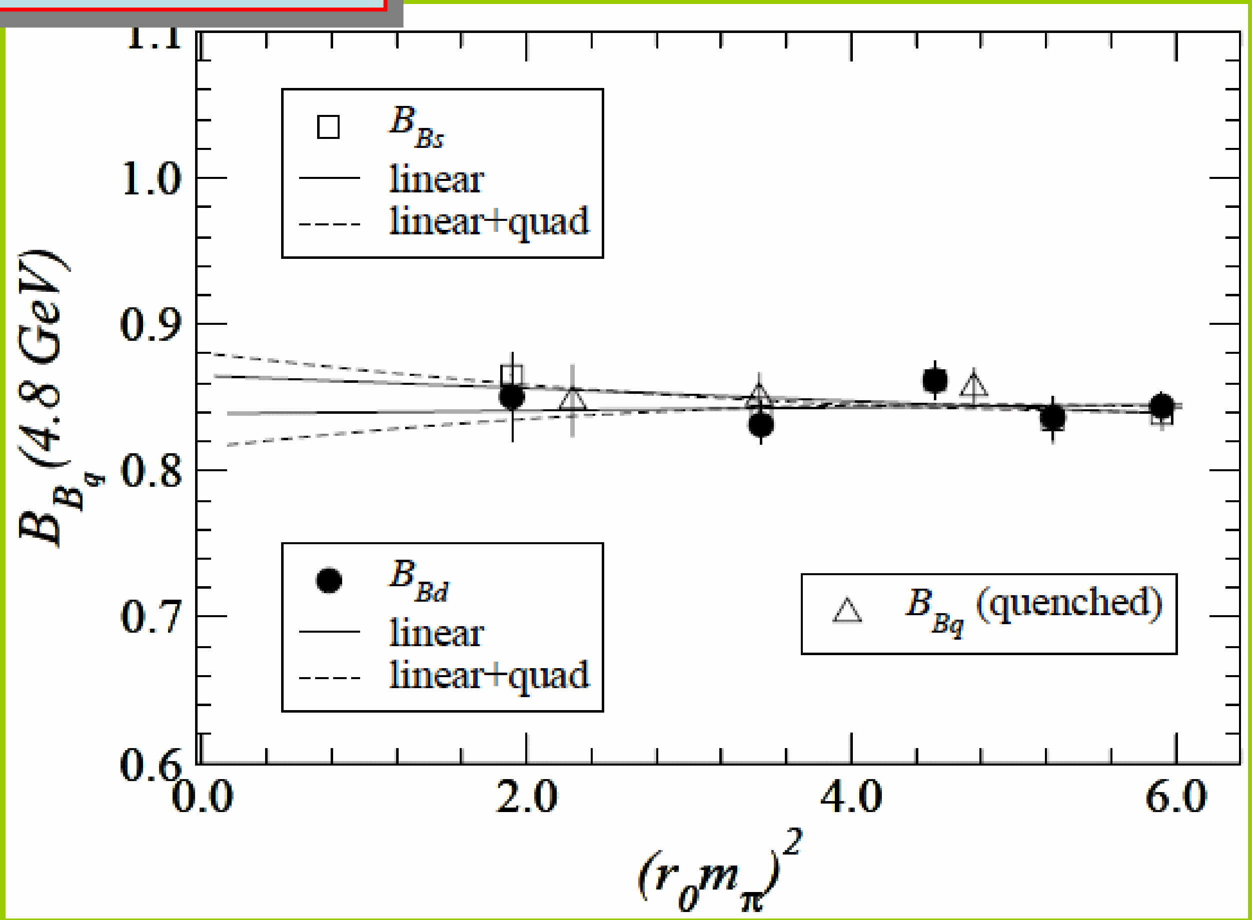
JLQCD(03), $N_F=2$

B-$\bar{B}$ MIXING ON THE LATTICE

$$\langle \overline{B}_q | (\bar{b}\gamma_L^\mu q)(\bar{b}\gamma_L^\mu q) | B_q \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$

Chiral Loops not a issue for $\hat{B}_{B_d}$

They are expected small from ChPT



Lattice data are consistent with a constant.

B mixing from Lattice QCD

	Lellouch, ICHEP 2002	Hashimoto, ICHEP 2004
f_B (MeV)	203(27)(+0-20)	189(27)
f_{B_s} (MeV)	238(31)	230(30)
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ (MeV)	276(38)	262(35)
f_{B_s} / f_B	1.18(4)(+12-0)	1.22(+5-6)
ξ	1.18(4)(+12-0)	1.23(6)

Now averages include rough ``estimates'' of chiral log ($m_q/m_s>0.2$) and unquenched effects ($N_F=2+1$)

Summarizing from Lattice QCD

Observable	$\lvert\varepsilon\rvert$	ΔM_{B_s}	$\dfrac{\Delta M_{B_s}}{\Delta M_{B_d}}$	$B \to \left(\begin{smallmatrix}\pi\\ \rho\end{smallmatrix}\right) l\nu$	$B \to \left(\begin{smallmatrix}D\\ D^*\end{smallmatrix}\right) l\nu$
CKM	Im$[V_{td}]$	$\lvert V_{ts}\rvert^2$	$\lvert V_{ts}\rvert^2/\lvert V_{td}\rvert^2$	$\lvert V_{ub}\rvert^2$	$\lvert V_{cb}\rvert^2$
Matr. Elem.	$\hat{B}_K$	$f^2_{B_s}\hat{B}_{B_s}$	$\dfrac{f^2_{B_s}\hat{B}_{B_s}}{f^2_{B_d}\hat{B}_{B_d}}$	$\lvert\langle \begin{smallmatrix}\pi\\ \rho\end{smallmatrix} \lvert J_L^{ub}\rvert B\rangle\rvert^2$	$\lvert\langle \begin{smallmatrix}D\\ D^*\end{smallmatrix} \lvert J_L^{cb}\rvert B\rangle\rvert^2$
Err. in Quen.	7%	10%	6%	30%	8%
Error	15%	25%	10%	-	3%
Proj. error	5%	10%	5%	20%	6%

At present

- If we are able to run full QCD simulations with machines $\mathcal{O}(1-10)$ Tflops

$$L = 2.0 - 2.5\,\text{fm} \qquad \frac{m_\pi}{m_\rho} = 0.25 - 0.5 \qquad a = 0.05 - 0.10\,\text{fm}$$

- **<u>Expected results in coming year and so:</u>**

Staggered, TmQCD … fermions… at more lattice spacings (continuum limit) and smaller masses.

B- Mixing from QCD Sum Rules

$f_B = 210(19)\,\mathrm{MeV}$,

$f_{B_s} = 244(21)\,\mathrm{MeV}$ (Jamin et al '02)

$\hat{B}_B = 1.60(3)$ (Körner et al '03)

Non-perturbative effects parameterized by a series of power corrections, with coefficients proportional to various ``unknown'' condensates

f_B $O(\alpha_s^2)$ recently calculated

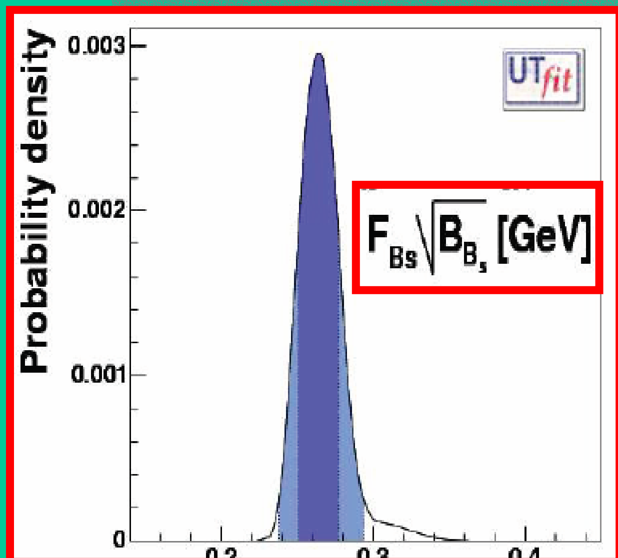
B_B $O(\alpha_s)$ completed but non-factorisable contributions important

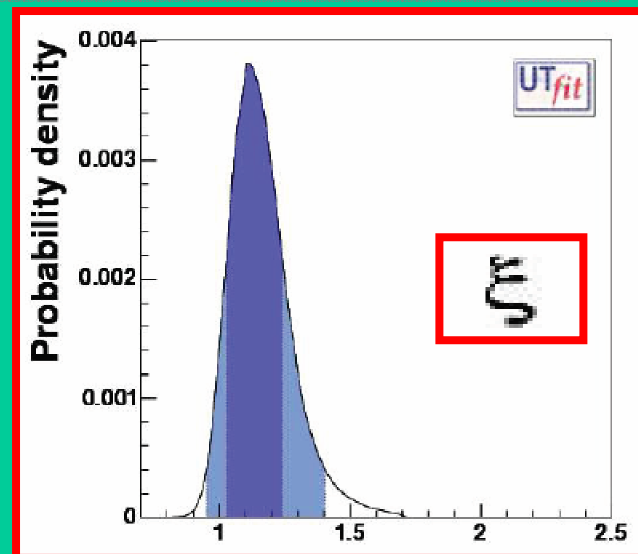
Problem: too many parameters, loosely constrained, which thus reduce the power of this method for a high precision era.

At least for the time being, we are happy to know that

Lattice QCD vs UT FITS

(Cross-Checks)





	LATTICE QCD	UTFIT
$f_{B_s}\sqrt{\hat{B}_{B_s}}$	(262 ± 35)MeV	(265 ± 13)MeV
ξ	1.23 ± 0.06	1.15 ± 0.11

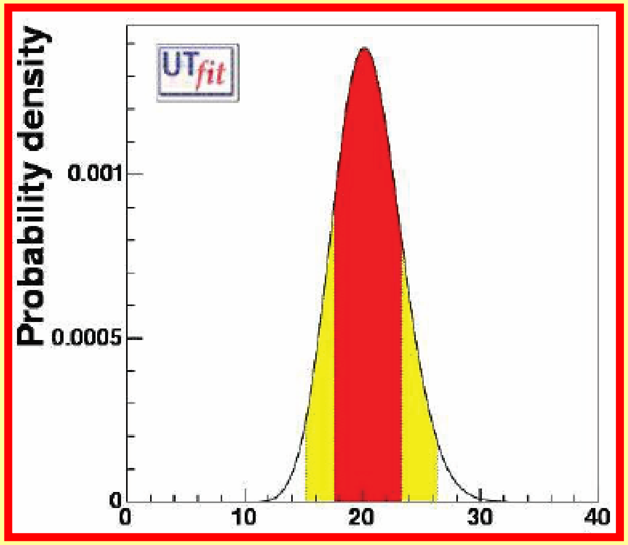
by courtesy of V. Lubicz and UTFit

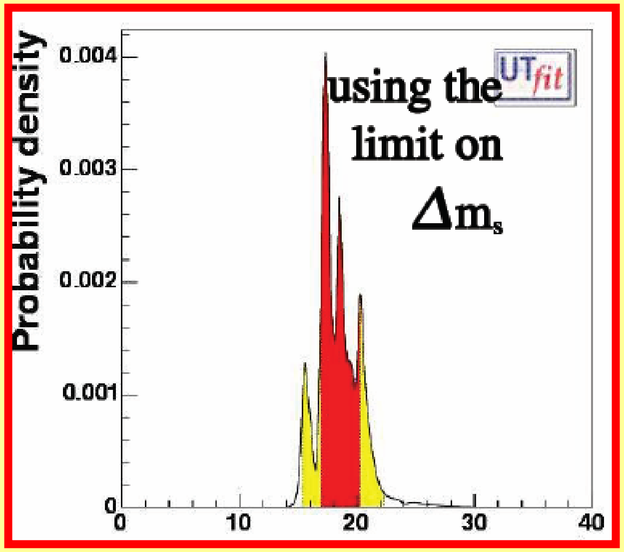
Moreover, we can predict

by courtesy of V. Lubicz and UTFit c.

Prediction for Δm_s

Δm_s NOT USED	WITH ALL CONSTRAINTS





$\Delta m_s = (20.4 \pm 2.8)\ ps^{-1}$	$\Delta m_s = (20.4 \pm 2.8)\ ps^{-1}$

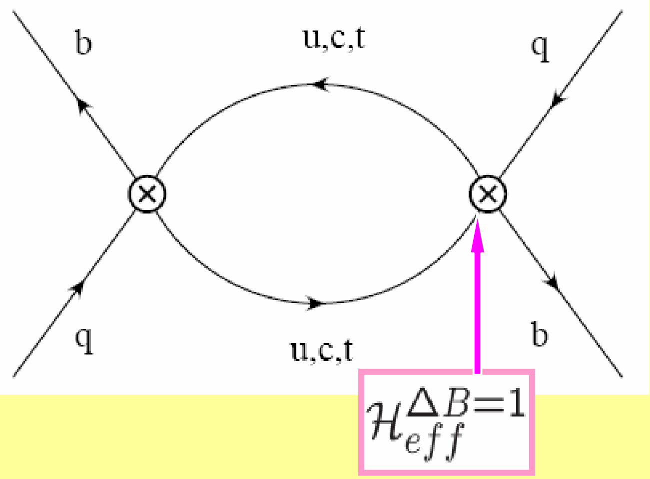
A measurement is expected at FERMILAB

2. Width Differences: $\Delta\Gamma_s$, $\Delta\Gamma_d$

- Heavy Quark Expansion ($m_b \to \infty$)
- $\Delta B=2$ Matrix Elements (mainly quenched!)
- Theoretical Predictions

Theoretical Calculation

$$\Delta\Gamma_q = -2\,|M_{21}^q|\,\mathrm{Re}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$



- $M_{21}^q \rightarrow$ **real part of the box diagram**:

Top contribution dominant$\Rightarrow$ OPE ($m_t \rightarrow \infty$, $m_W \rightarrow \infty$)

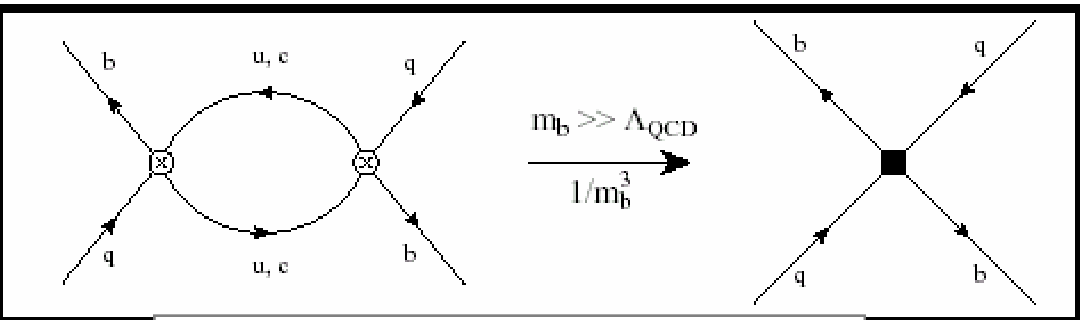
$$M_{21}^q = \frac{G_F^2 M_W^2}{(4\pi)^2}(V_{tb}^* V_{tq})^2 S(x_t)\,\frac{\langle \overline{B}_q|(\bar{b}\gamma_L^\mu q)(\bar{b}\gamma_L^\mu q))|B_q\rangle}{2M_{B_q}},$$

OPE

$$\Delta m_q = 2\,|M_{21}^q|$$

- $\Gamma_{21}^q \rightarrow$ **"imaginary" part of the box diagram**:

Only c and u quarks contribute$\Rightarrow$ OPE ($m_W \rightarrow \infty$, $m_b \rightarrow \infty$



Large energy release: $q^2 \sim m_b^2$

OPE+HQE

$$(m_b >> \Lambda_{QCD})$$

$\Delta B = 2$ Heavy Quark Expansion

$$\Gamma_{21}^q = \frac{1}{2M_{B_q}} \text{Disc}\langle \overline{B}_q | i \int d^4x \, T\left(\mathcal{H}_{eff}^{\Delta B=1}(x)\mathcal{H}_{eff}^{\Delta B=1}(0)\right) |B_q\rangle$$

$$\stackrel{m_b\to\infty}{\to} = -\frac{G_F^2 m_b^2}{24\pi M_{B_q}} \left[\sum_{i=1,2} c_i^q(\mu_2) \langle \overline{B}_q | \mathcal{O}_i^q(\mu_2) | B_q\rangle + \delta_{1/m}^q \right],$$

- $\vec{c}_k^{\,q}(\mu) = \vec{A}_k^{\,q}(m_c) + \frac{\alpha_s}{4\pi} \vec{B}_k^{\,q}(\mu, m_c) + \ldots$

contain physics from scales $\geq \mu = \mathcal{O}(m_b)$

Computable in perturbation theory:

$\star \vec{A}_k^{\,q}(m_c)$ including $1/m_b$ terms calculated in

M. Beneke *et al.*, Phys. Rev. D54, 4419, [hep-ph/9605259]

A.S. Dighe *et al.*, Nucl. Phys. B625, 377,[hep-ph/0109088]

$\star \vec{B}_k^{\,q}(\mu, m_c)$ at leading order in $1/m_b$ calculated in

M. Beneke *et al.*, Phys. Lett. B459, 631 [hep-ph/9808385]

M. Beneke *et al.*, [hep-ph/0307344]

M. Ciuchini *et al.*, JHEP 0308:031,2003,[hep-ph/0308029]

$\mathcal{O}_1^q = (\bar{b}\gamma_L^\mu q)(\bar{b}\gamma_L^\mu q)$, $\mathcal{O}_2^q = (\bar{b}Lq)(\bar{b}Lq)$

$\delta_{1/m}^q$ denotes the sub-leading $1/m_b$ corrections.

Long-distance physics

Expansion of Γ_{21} in $1/m_b$ and α_s,

by assuming Quark-Hadron duality

$\Delta B = 2$ Operator Matrix Elements

Leading contribution in $1/m_b$

$$\mathcal{O}_1^q = (\bar{b}\gamma_L^\mu q)\,(\bar{b}\gamma_L^\mu q) \leftrightarrow B_1^q, \quad \mathcal{O}_2^q = (\bar{b}Lq)\,(\bar{b}Lq) \leftrightarrow B_2^q.$$

B_q^1, B_q^2: from the LATTICE with different methods

or QCD *sum rules* [J.G. Korner et al., 2003]

$B_q^1 \doteq \widehat{B}_{B_d}$

(see Δm_q)

Subleading contribution ($1/m_b$ terms)

$$R_1^q = \frac{m_q}{m_b}(\bar{b}_i L q_i)(\bar{b}_j R q_j) \leftrightarrow B_3^q\,,$$

$$R_2^q = \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho q_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) q_j)\,,$$

$$R_3^q = \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho q_i)(\bar{b}_j (1-\gamma_5) q_j)\,,$$

$$R_4^q = \frac{1}{m_b}(\bar{b}_i (1-\gamma_5) i D_\mu q_i)(\bar{b}_j \gamma^\mu (1-\gamma_5) q_j)\,.$$

R_1^q, R_4^q: related, through Fierz and eq. of motion, to operators computed on the LATTICE

$\star$ M. Ciuchini *et al.*, JHEP 0308:031,2003,[hep-ph/0308029]

R_2^q, R_3^q: from the VSA

B_1^q, B_2^q from the lattice

- HQET: static limit $(m_b \to \infty)$

$$B_1^s = 0.83(5)(6),\ B_2^s = 0.81(2)(10)$$

[V. Gimènez e J. Reyes, 2000]

- QCD: relativistic heavy quark

$(m_c \lesssim m_Q < m_b,\ m_Q \to m_b)$

$$B_1^s = 0.91(3)^{+0}_{-6},\ B_2^s = 0.86(2)^{+2}_{-3}$$

[APE (D. Becirevic et al.), 2000]

- QCD + HQET ★

$$B_1^s = 0.87(2)(5),\ B_2^s = 0.84(2)(4)$$

[APE (D. Becirevic et al.), 2001]

- NRQCD: by including corrections of $\mathcal{O}(1/m_b)$

$$B_1^s = 0.85(3)(11),\ B_2^s = 0.82(2)(11)$$

[Hi-KEK (S. Hashimoto et al.), 2000]

Qhenched

- NRQCD "unquenched": $n_f = 2$

$$B_1^s = 0.85(2)(6),\ B_2^s = 0.84(6)(8)$$

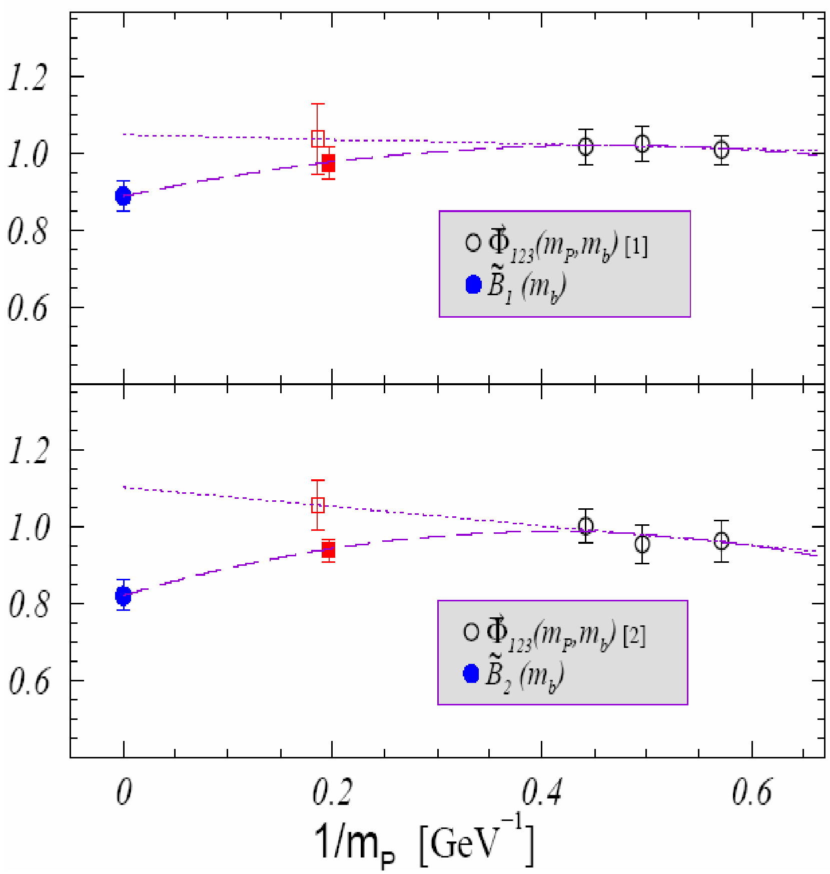
[JLQCD (S. Aoki et al.), 2001-2003]

Values ren. in $\overline{\text{MS}}$ at scale m_b

- Values at one lattice spacing
- No Continuum Extrapolation

QCD + HQET

SPQcdR, 2001



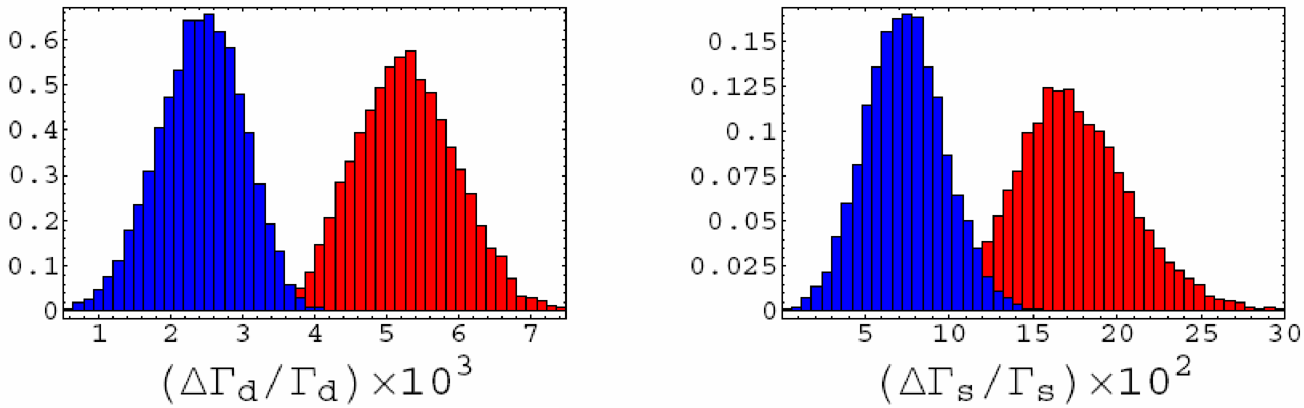
- Combine the static HQET results for B-parameters with the relativistic lattice QCD ones
  $\Rightarrow$ extrapolation $\rightarrow$ "interpolation"

- Perturbative matching of the anomalous dimensions of 4-f QCD and HQET operators made @ NLO in perturbation theory!

- So far, the approach has been just applied to the quenched case

Width Differences

NLO distr. vs LO distr.



Theoretical predictions

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (2.42 \pm 0.59)10^{-3} \qquad \frac{\Delta\Gamma_s}{\Gamma_s} = (7.4 \pm 2.4)10^{-2}$$

[M. Ciuchini, E. Franco, V. Lubicz, F. M and C. Tarantino, 2003]

- *NLO-LO difference is remarkable*
- Difficulty to reduce the theor. error (from m_b and rin. scale)

Experimental measurements:

$\Delta\Gamma_s / \Gamma_s = 0.35^{+0.12}_{-0.16}$ HFAG, 05

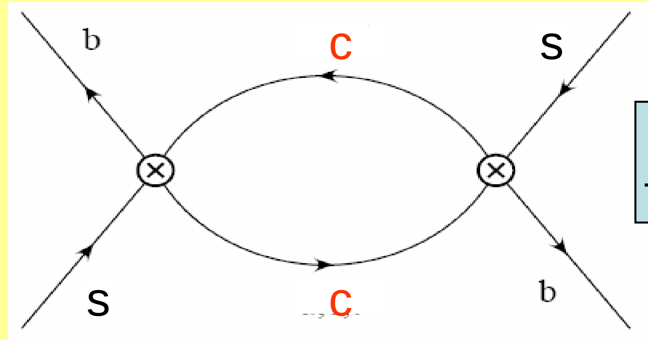
$\Delta\Gamma_d / \Gamma_d = 0.008 \pm 0.037 \pm 0.018$ Babar, 03

The most recent

$\Delta\Gamma_s / \Gamma_s = 0.65^{+0.25}_{-0.33} \pm 0.01$ CDF, 05

$\Delta\Gamma_s / \Gamma_s = 0.21^{+0.27}_{-0.40}$ D0, 05 prel.

$\Delta\Gamma_s/\Gamma_s$ and the HQE



CKM favoured

$$\frac{\Delta\Gamma_s}{\Gamma_s} \propto \left[(c_1^{(0)} + c_1^{(1)})\langle O_1^s \rangle + (c_2^{(0)} + c_2^{(1)})\langle O_2^s \rangle + \delta_{1/m_b} \right]$$
$$= \{(0.018 - 0.014) + (0.240 - 0.064) - 0.097\}$$

HQE	$\Delta\Gamma_s/\Gamma_s$
LO	26%
LO $+O(1/m_b)$	16%
NLO	18%
NLO $+O(1/m_b)$	7%

$$c_1 << c_2$$

$$\frac{c_2^{(1)}}{c_2^{(0)}} = -27\%$$

(large cancelations at the NLO-QCD)!!

$$\frac{\delta_{1/m_b}}{c_2\langle O_2^s \rangle} = -55\%$$

(large cancelations at $O(1/m_b)$)

Let me remind you, that in any SM extensions

$$\left(\frac{\Delta\Gamma_s}{\Gamma_s}\right)^{New\ Phy.} \leq \left(\frac{\Delta\Gamma_s}{\Gamma_s}\right)^{SM}$$

2. Lifetime ratios: $\tau(B^+)/\tau(B_d), \tau(\Lambda_b)/\tau(B_d)$

- Heavy Quark Expansion ($m_b \to \infty$)
- $\Delta B=0$ Matrix Elements (only quenched!)
- Results

Theoretical Calculation

The optical theorem gives

$$\Gamma_{H_b} = \frac{1}{m_{H_b}} Im \left[\langle H_b | \mathcal{H}^{\Delta B=0} | H_b \rangle \right]$$

where

$$\mathcal{H}^{\Delta B=0} = i \int d^4x\, T \left[\mathcal{H}^{\Delta B=1}(x) \mathcal{H}^{\Delta B=1}(0) \right]$$

NON-LOCAL OPERATOR

HQE allows us to write the rate as a series of local operators

$$\Gamma = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3 (2M_B)} \left[c^{(3)} \langle \bar{b}b \rangle + c^{(5)} \frac{g_s}{m_b^2} \langle \bar{b}\sigma_{\mu\nu} G^{\mu\nu} b \rangle + \frac{96\pi^2}{m_b^3} \sum_k \left(c_k^{(6)} \langle O_k^{(6)} \rangle + \frac{c_k^{(7)}}{m_b} \langle O_k^{(7)} \rangle \right) \right]$$

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.00 - \Delta_{\rm spec}^{B^+}, \qquad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.98(1) - \Delta_{\rm spec}^{\Lambda},$$

$$\Delta_{spec}^{H_b} = \frac{16\pi^2}{m_b^3}\left(c_k^{(6)} \left(\langle O_k^q \rangle_{H_b} - \langle O_k^q \rangle_{B_d} \right) + \frac{c_k^{(7)}}{m_b} \delta_{1/m} \right)$$

1. Up to $1/m_b^2$, no spectator effects and all b-hadrons have the same lifetime
2. Spectator effects appear first at order $1/m_b^3$, enhanced by a $16\pi^2$

Wilson Coefficients calculated in PT: $c_k^{(6)}$ @ NLO, $c_k^{(7)}$ @ LO

Bigi, Uraltsev (94), M. Neubert and C.T. Sachrajda (97) $O(1/m_b^3)$

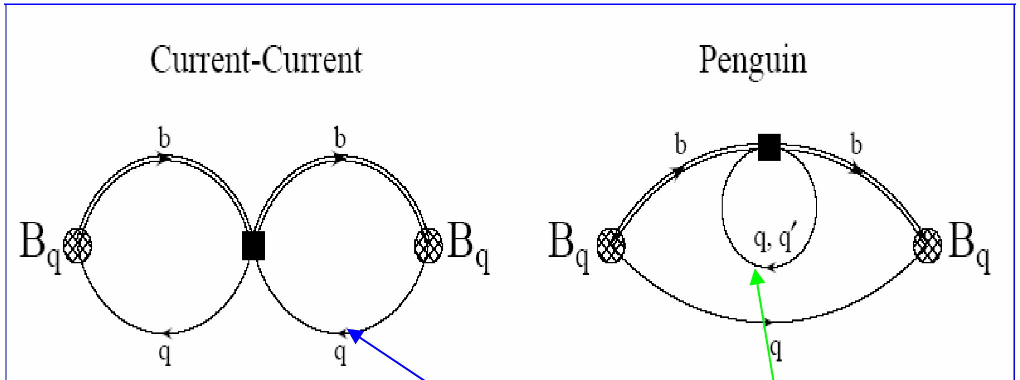
Ciuchini, Franco, Lubicz, F. M (01) $O(\alpha_s/m_b^3)$

Franco, Lubicz, F. M and Tarantino (02)

Beneke, Buchalla, Greub, Lenz and Nierste (02) $O(m_c \alpha_s/m_b^3)$

Gabbiani, Onishchenko and Petrov (04) $O(1/m_b^4)$

B-Meson Matrix Elements



At lowest order in QCD, spectator effects are defined by 4 current-current operators:

$$\mathcal{O}_1^q = (\bar{b}\gamma_L^\mu q)\,(\bar{q}\gamma_L^\mu b)$$
$$\mathcal{O}_2^q = (\bar{b}Lq)\,(\bar{q}Rb)$$
$$\mathcal{O}_3^q = (\bar{b}\gamma_L^\mu t^a q)\,(\bar{q}\gamma_L^\mu t^a b)$$
$$\mathcal{O}_4^q = (\bar{b}Lt^a q)\,(\bar{q}Rt^a b)\,.$$

$$\frac{\langle B_q|\mathcal{O}_{1,2}^q|B_q\rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2}\left(B_{1,2}^q + \delta_{1,2}^{qq}\right) \qquad \frac{\langle B_q|\mathcal{O}_{3,4}^q|B_q\rangle}{2M_{B_q}} = \frac{f_{B_q}^2 M_{B_q}}{2}\left(\varepsilon_{1,2}^q + \delta_{3,4}^{qq}\right)$$

1. So far, only estimates for the current-current contributions
2. Penguin operators non computed

However Penguin contributions cancel in $\tau(B^+)/\tau(B_d)$, due to SU(2) symmetry

$\Delta B=0$ B-parameters

No Unquenched Calculation yet

• **Lattice-HQET** $(m_b \to \infty)$

$$B_1^d = 1.06 \pm 0.08, \quad B_2^d = 1.01 \pm 0.07,$$
$$\varepsilon_1^d = -0.01 \pm 0.03, \quad \varepsilon_2^d = -0.03 \pm 0.02.$$

[M. Di Pierro and C.T. Sachrajda, 1998]

• **Lattice-QCD** $(m_c \lesssim m_Q < m_b,\ m_Q \to m_b)$

$$B_1^d = 1.2 \pm 0.2, \quad B_2^d = 0.9 \pm 0.1,$$
$$\varepsilon_1^d = 0.04 \pm 0.01, \quad \varepsilon_2^d = 0.04 \pm 0.01.$$

[APE (D. Becirevic et al)., 2001]

• **Sum Rules, in HQET**

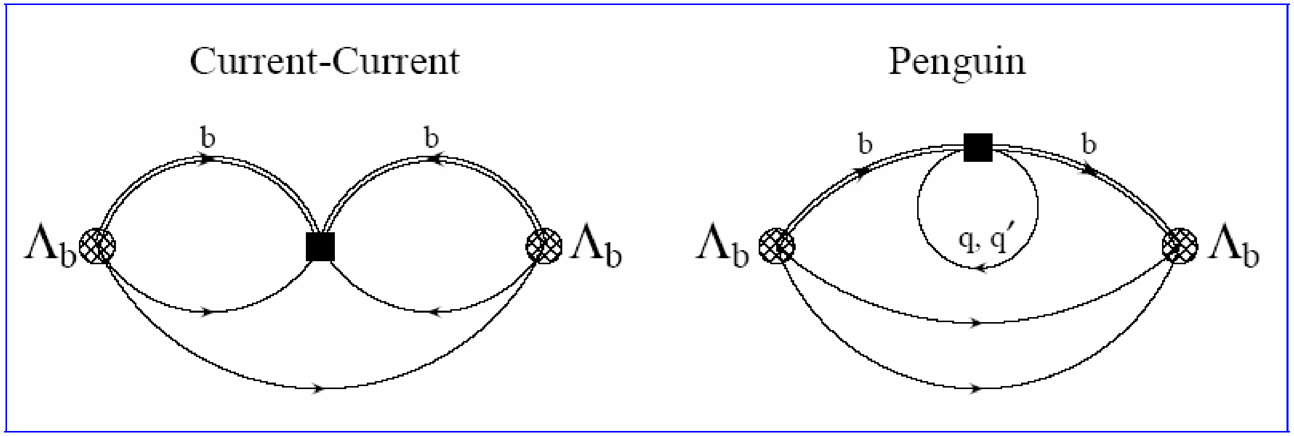
$$B_1^d = 1.01 \pm 0.01, \quad B_2^d = 0.99 \pm 0.01,$$
$$\varepsilon_1^d = -0.08 \pm 0.02, \quad \varepsilon_2^d = -0.01 \pm 0.03.$$

[M.S. Baek et al., 1998]

Subleading spectator-effect contribution $O(1/m_b^4)$

8 operators, from the *VSA*

Baryon Matrix Elements



$$\frac{\langle \Lambda_b | \mathcal{O}_1^q | \Lambda_b \rangle}{2M_{\Lambda_b}} = \frac{f_B^2 M_B}{2} \left(L_1 + \delta_1^{\Lambda q} \right), \quad (q = u, d)$$

$$\frac{\langle \Lambda_b | \mathcal{O}_3^q | \Lambda_b \rangle}{2M_{\Lambda_b}} = \frac{f_B^2 M_B}{2} \left(L_2 + \delta_2^{\Lambda q} \right) \quad (q = u, d)$$

- At present, L_i are available only from an exploratory lattice calculation
  M.Di Pierro et al, hep-lat/9906031

- Subleading contributions: **8 operators, from *quark-diquark model***

- δ-contributions do not cancel in $\tau(\Lambda_b)/\tau(B_d)$!!!

Theoretical Predictions

HQE	$\frac{\tau(B^+)}{\tau(B_d)}$	$\frac{\tau(B_s)}{\tau(B_d)}$	$\frac{\tau(\Lambda_b)}{\tau(B_d)}$
LO	**1.01(3)**	**1.00(1)**	**0.93(4)**
NLO	**1.06(3)**	**1.00(1)**	**0.90(5)**
NLO+ $O(1/m_b^4)$	**1.06(2)**	**1.00(1)**	**0.88(5)**

Lifetime ratio	Measured value
$\tau(B^+)/\tau(B^0)$	1.076 ± 0.008
$\overline{\tau}(B_s^0)/\tau(B^0)^a$	0.920 ± 0.030
$\tau(\Lambda_b^0)/\tau(B^0)$	0.806 ± 0.047
$\tau(b\text{-baryon})/\tau(B^0)$	0.792 ± 0.032

[a] Using $\overline{\tau}(B_s^0) = 1/\Gamma_s = 2/(\Gamma_L + \Gamma_H)$.

HFAG, **05**

- **Good agreement between Theory and Experiment**

$(\tau(\Lambda_b)/\tau(B_d)$ at $1\sigma)$

- lattice results are preliminary, (*M.Di Pierro et al. 1999*)
- Penguin contributions should be computed,

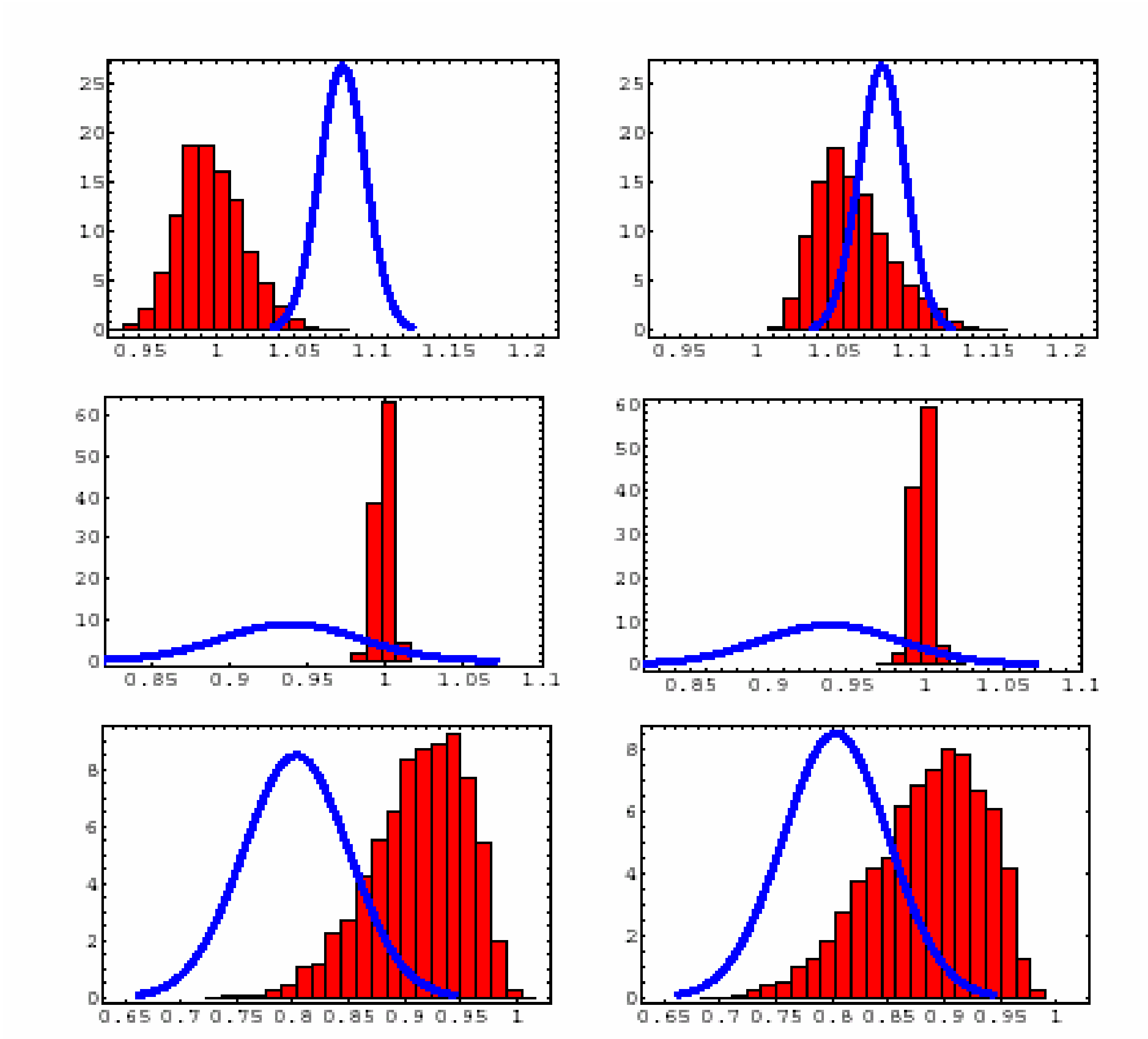
Lifetime Ratios

LO **NLO** $+O(1/m_b^4)$

$$\frac{\tau(B^+)}{\tau(B_d)}$$

$$\frac{\tau(B_s)}{\tau(B_d)}$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}$$



Conclusions

- A hard job has been made on B-flavour physics from theorists, using QCD Sum Rules and Lattice QCD;
- So far, no significant deviations from the SM observed!
- Then, New physics is hidden in the error bars!!
- Mind: Susy is affected by hadronic quantities as much as the SM

Future Favour Physics Programme:

refining estimates of hadronic uncertainties by Lattice QCD:

Realistic unquenched studies with finer lattices and lighter quark masses.

not at all easy!! neither fast!!

BACKUP

SEARCH FOR NEW PHYSICS

2) "Given the present theoretical and experimental constraints, to which extent the UTA can still be affected by New Physics contributions?"

An interesting case:

New Physics in B_d–$\overline{B}_d$ mixing

The New Physics mixing amplitudes can be parameterized in a simple general form:

$$M_d = C_d \, e^{2i\varphi_d} (M_d)^{SM}$$

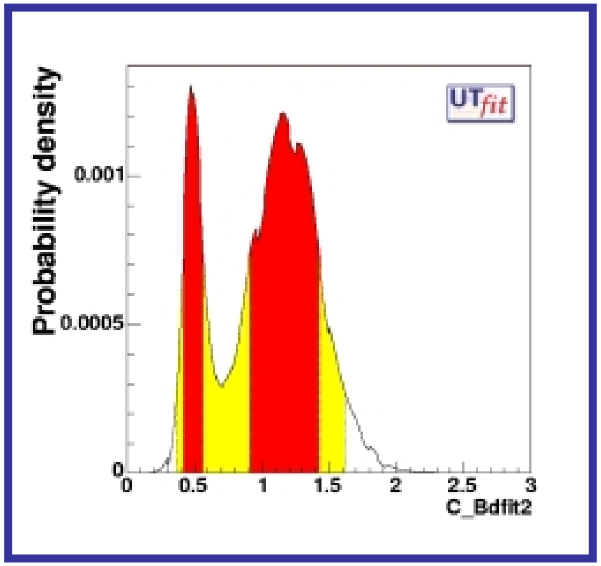
→

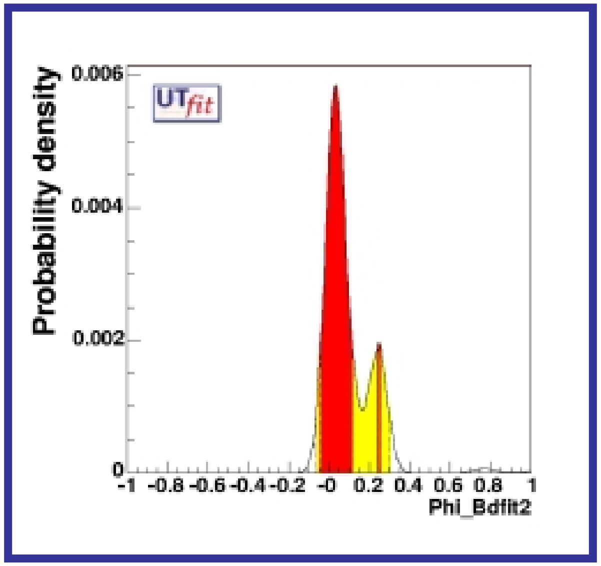
$$\Delta m_d = C_d \, (\Delta m_d)^{SM}$$

$$A(J/\psi \, K_S) \sim \sin 2(\beta + \varphi_d)$$

TWO SOLUTIONS:

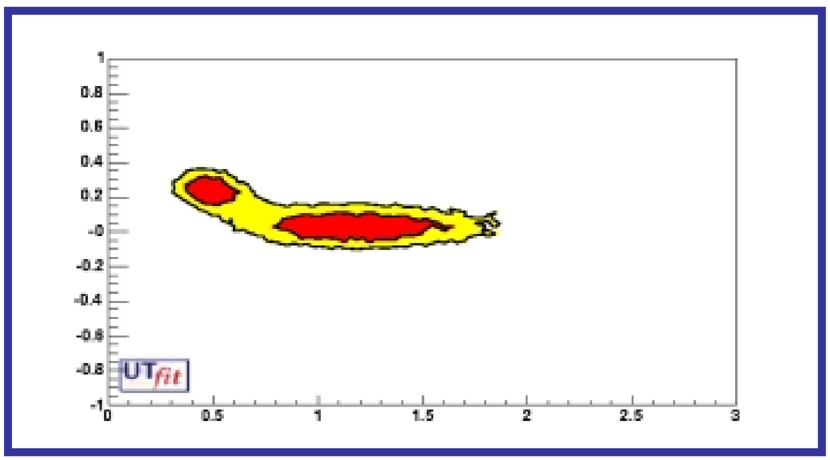
Standard Model solution:
$C_d = 1$ $\varphi_d = 0$

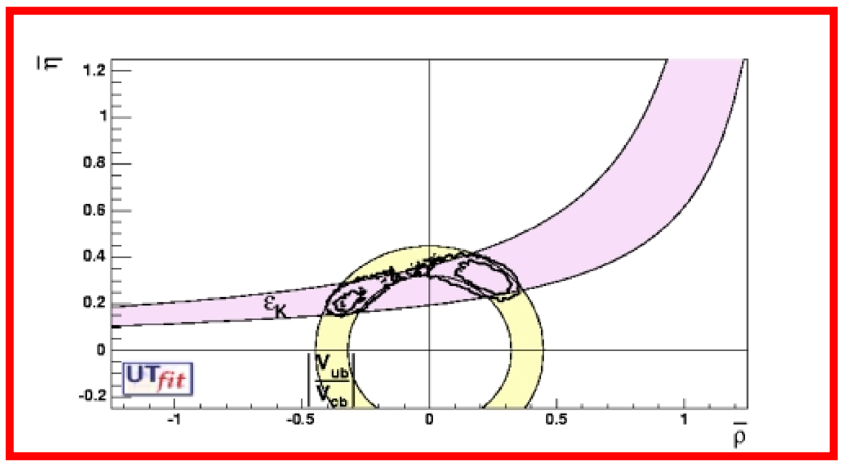




φ_d can be only determined up to a trivial twofold ambiguity:

$\beta+\varphi_d \rightarrow \pi-\beta-\varphi_d$





SUSY models

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i\, Q_i$$

$$\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i\, \bar{b}^j \gamma_\mu (1-\gamma_5) q^j\,, \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i\, \bar{b}^j (1-\gamma_5) q^j\,, \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j\, \bar{b}^j (1-\gamma_5) q^i\,, \\
O_4 &= \bar{b}^i (1-\gamma_5) q^i\, \bar{b}^j (1+\gamma_5) q^j\,, \\
O_5 &= \bar{b}^i (1-\gamma_5) q^j\, \bar{b}^j (1+\gamma_5) q^i\,,
\end{aligned}$$

For $M_S > m_t$ and with $\eta = \alpha_s(M_S)/\alpha_s(m_t)$

$$C_r(m_b^{pole}) = \sum_i \sum_s \left(b_i^{(r,s)} + \eta\, c_i^{(r,s)} \right) \eta^{a_i} \underbrace{C_s(M_S)}_{\delta \in C_s}$$

so far all δ's very small

SPQcdR 2001
(quenched)