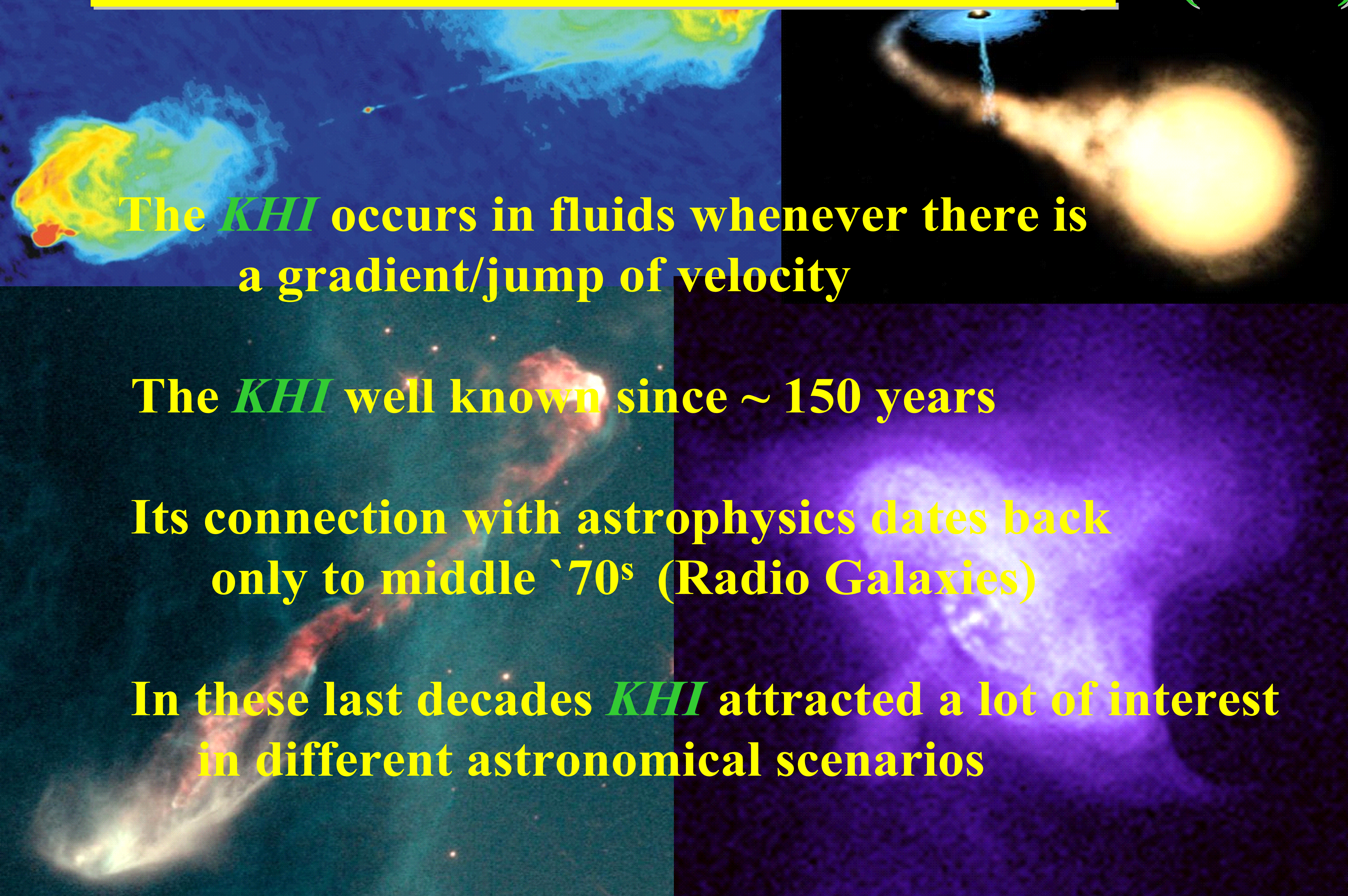
# ***The Kelvin Helmholtz Instability*** (*KHI*)



**The *KHI* occurs in fluids whenever there is a gradient/jump of velocity**

**The *KHI* well known since ~ 150 years**

**Its connection with astrophysics dates back only to middle `70s (Radio Galaxies)**

**In these last decades *KHI* attracted a lot of interest in different astronomical scenarios**

# Linear Analysis of the KHI

***Equilibrium***: a jet of radius $a$ moving in the $z$ direction
Perturbed with disturbances sinusoidal with $t$ and along $z$ and $\phi$ directions:

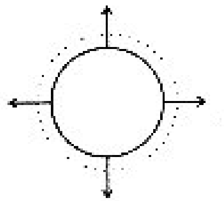
$$f(r) \exp [i(kz + n\phi - \omega t)]$$

(***normal modes***)

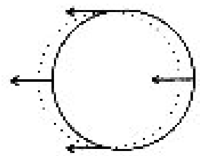
$k$: Longitudinal wavenumber

$n$: Azimuthal wavenumber

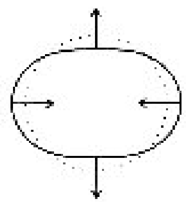
$f(r)$: $J_n(r)$, $H^o{}_n(r)$



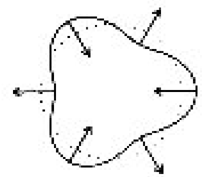
$n = 0$



$n = 1$



$n = 2$



$n = 3$

# *WHICH KIND OF ANALYSIS ?*

***Temporal***: $k$ real, $\omega$ complex: $t_i = 2\pi / \mathrm{Im}(\omega)$

***Spatial***: $\omega$ real, $k$ complex: $l_i = 2\pi / \mathrm{Im}(k)$

***Temporal*** $\leftrightarrow$ ***Spatial***: $l_i = t_i / v_g$, $v_g = d\,\mathrm{Re}(\omega) / d\,k$

➔ ***Dispersion relation:*** $\mathcal{D}(\Phi\ ;\ M,\ ka, \nu\ ,\ \ldots.) = 0$

**Nondimensional variables**

$$\Phi = \omega / ks, \quad M = v / s, \ \nu = \rho_{jet} / \rho_{ext}$$

# Linear KHI for adiabatic and HD jet

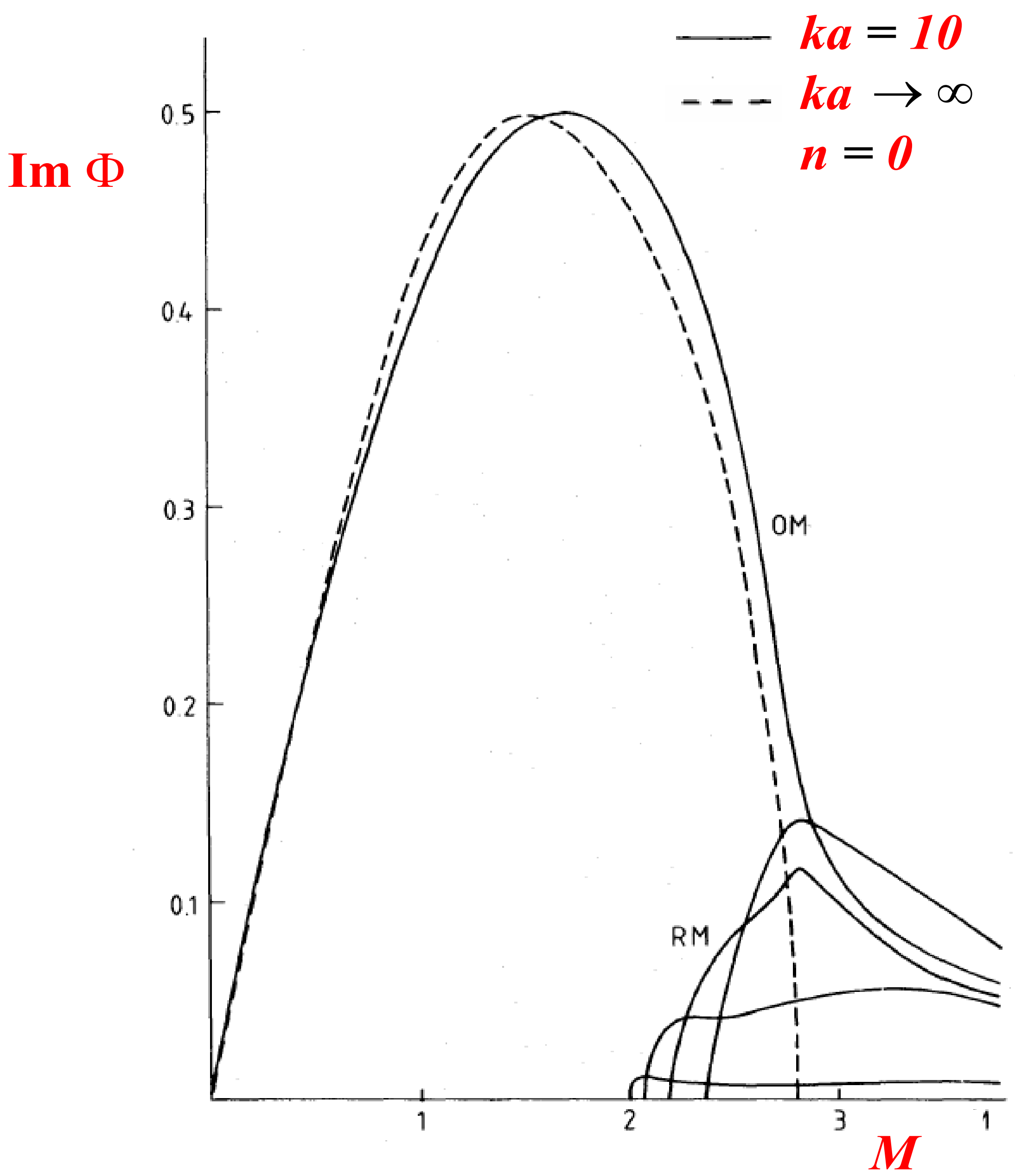
## *Two kinds of unstable perturbations*

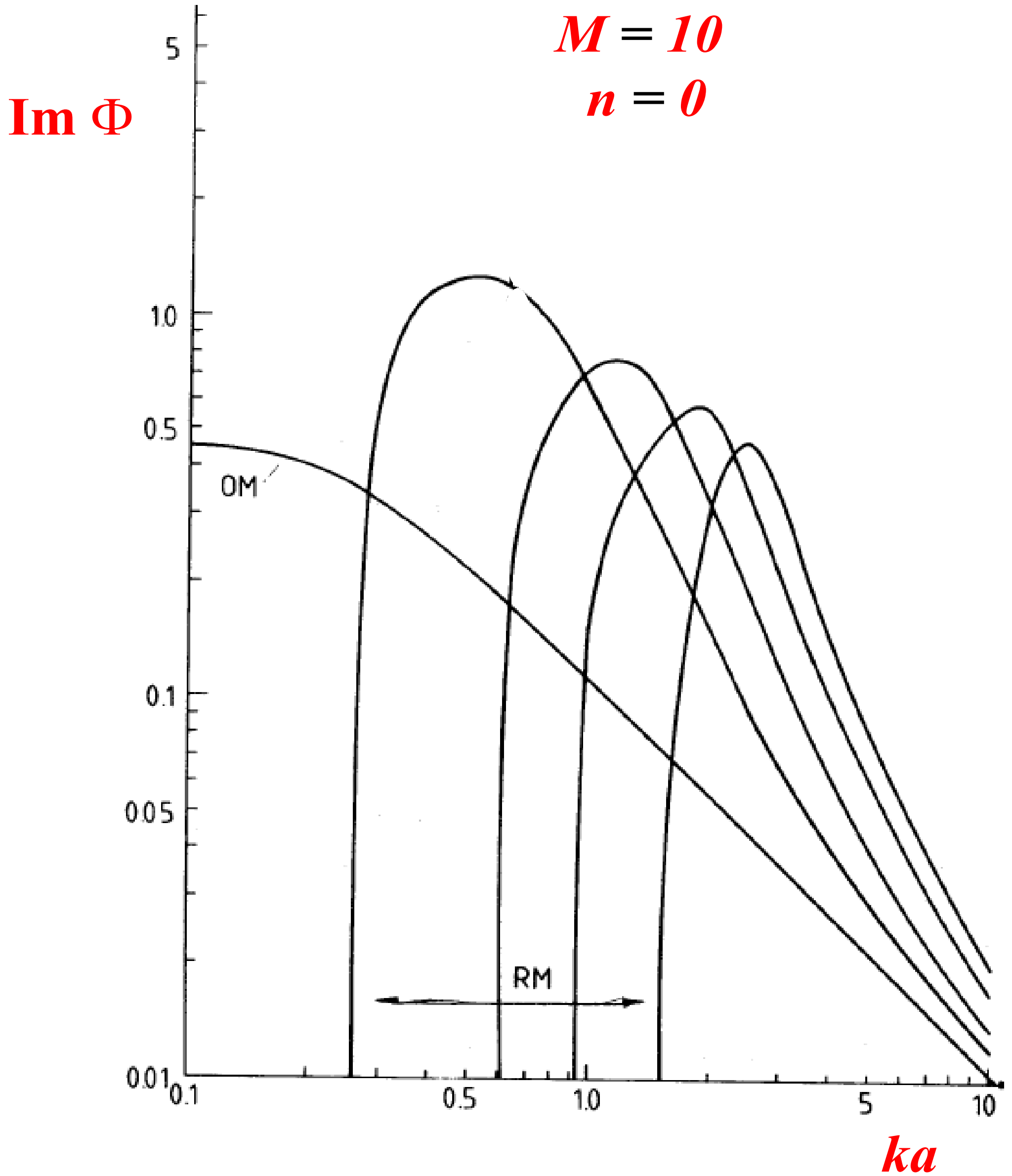
***1 - Ordinary (surface) modes*** ('*Historical*' *KHI* ):

- Oscillations of the interface
- Stabile only for ***ka*** ➔ $\infty$ and ***M*** $\geq$ ***2.8***

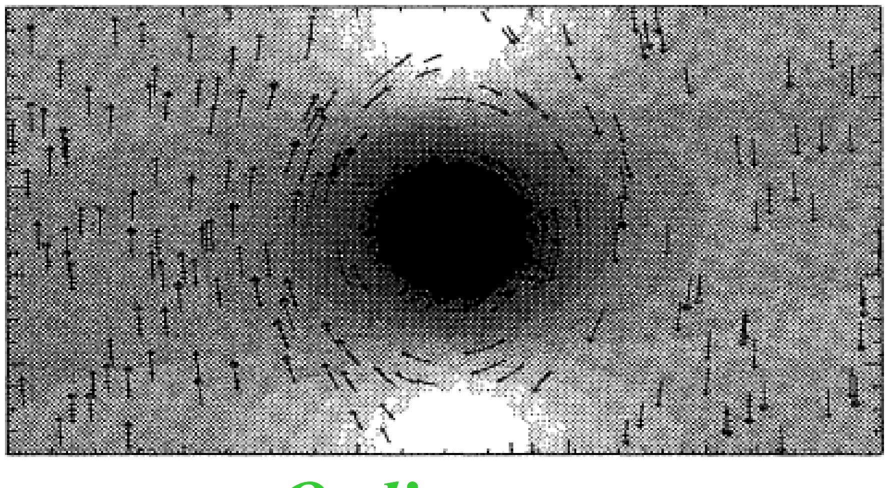
***2 - Reflected (body) modes***:

- ***Acoustic perturb.*** propagating inside and outside the jet
- Are found for ***M*** $>$ ***2*** (depending on ***ka***)
- Their onset related to the *over-reflection* of *sound waves* at the jet boundary


ka = 10
ka → ∞
n = 0
Im Φ
0.5
0.4
0.3
0.2
0.1
OM
RM
1
2
3
1
M

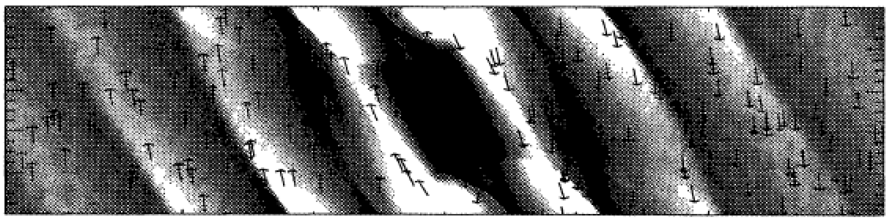

M = 10
n = 0
Im Φ
5
1.0
0.5
0.1
0.05
0.01
OM
RM
0.1
0.5
1.0
5
10
ka

## *Different evolution expected*
## *for ordinary and reflected modes*



*Ordinary*

***Steady vortex*** (*cat's eye*)



*Reflected*

*Acoustic waves* moving away from the trans. layer

*waves* ➔ *shocks*

# *Non linear Analysis of the KHI*

➔ Equilibrium: *A jet moving through the environment*
➔ At ***t*** = ***0*** unstable modes are switched on:

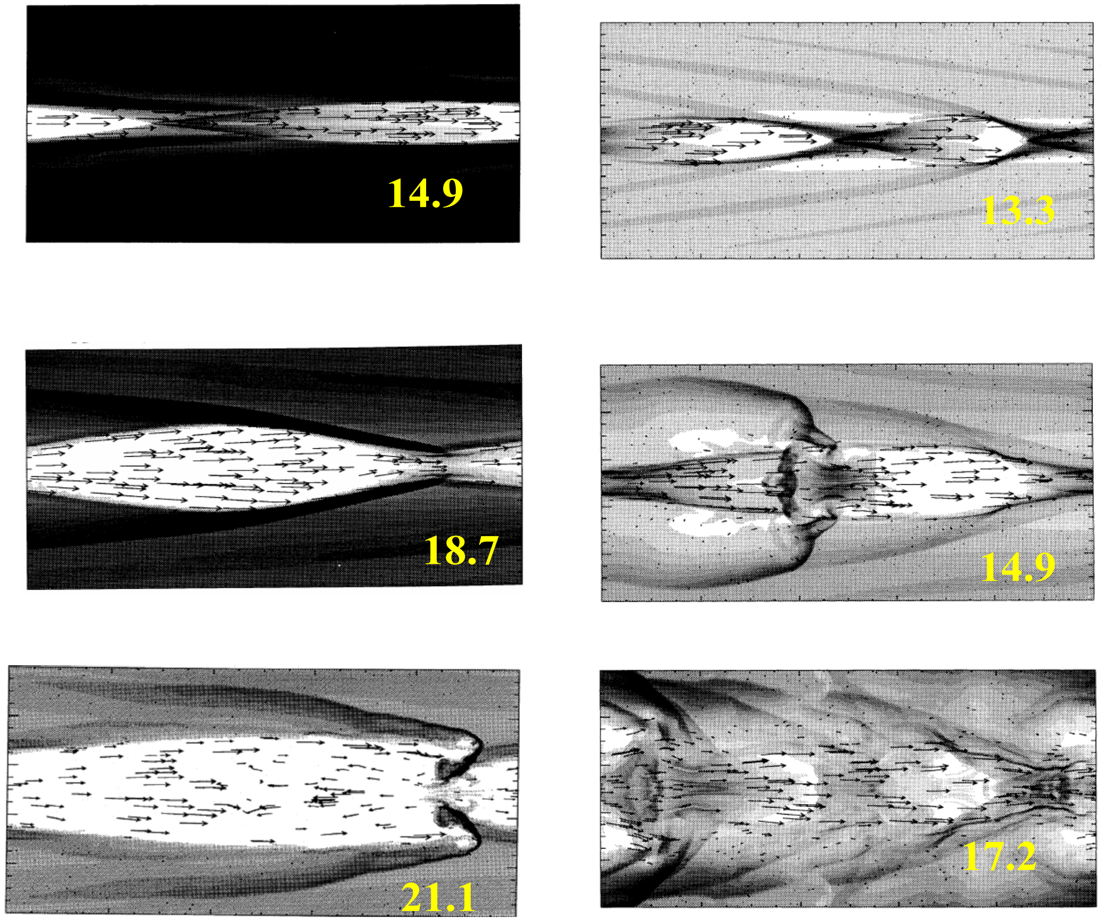
***Temporal***: A spectrum of ***ka*** assumed all along the flow
***Section of an infinite jet*** (***period. b.c***.)
**Long temporal evolutions**

***Spatial***: With a spectrum of $\Phi$ you perturb one side of the jet
***Free b.c. on the other side***
**Evolution of structures**
**advected along the flow**

## *2 - D / cyl.*

***M* = *20***

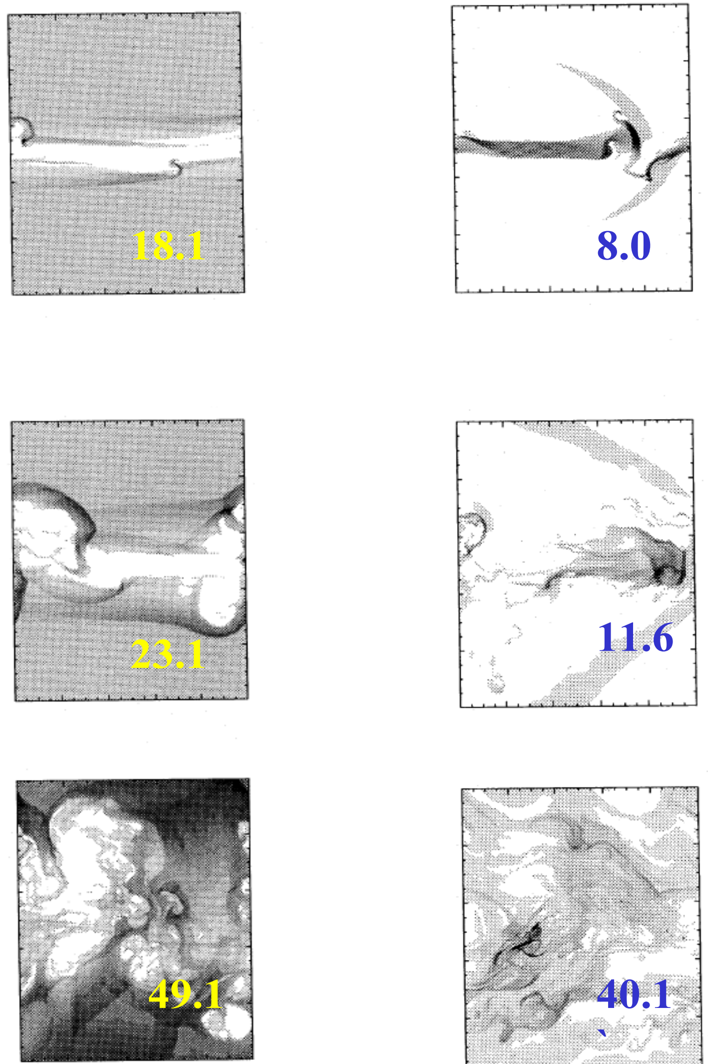
$\nu = 0.3$ $\nu = 3$




## *2 - D / slab*

***M* = *10***

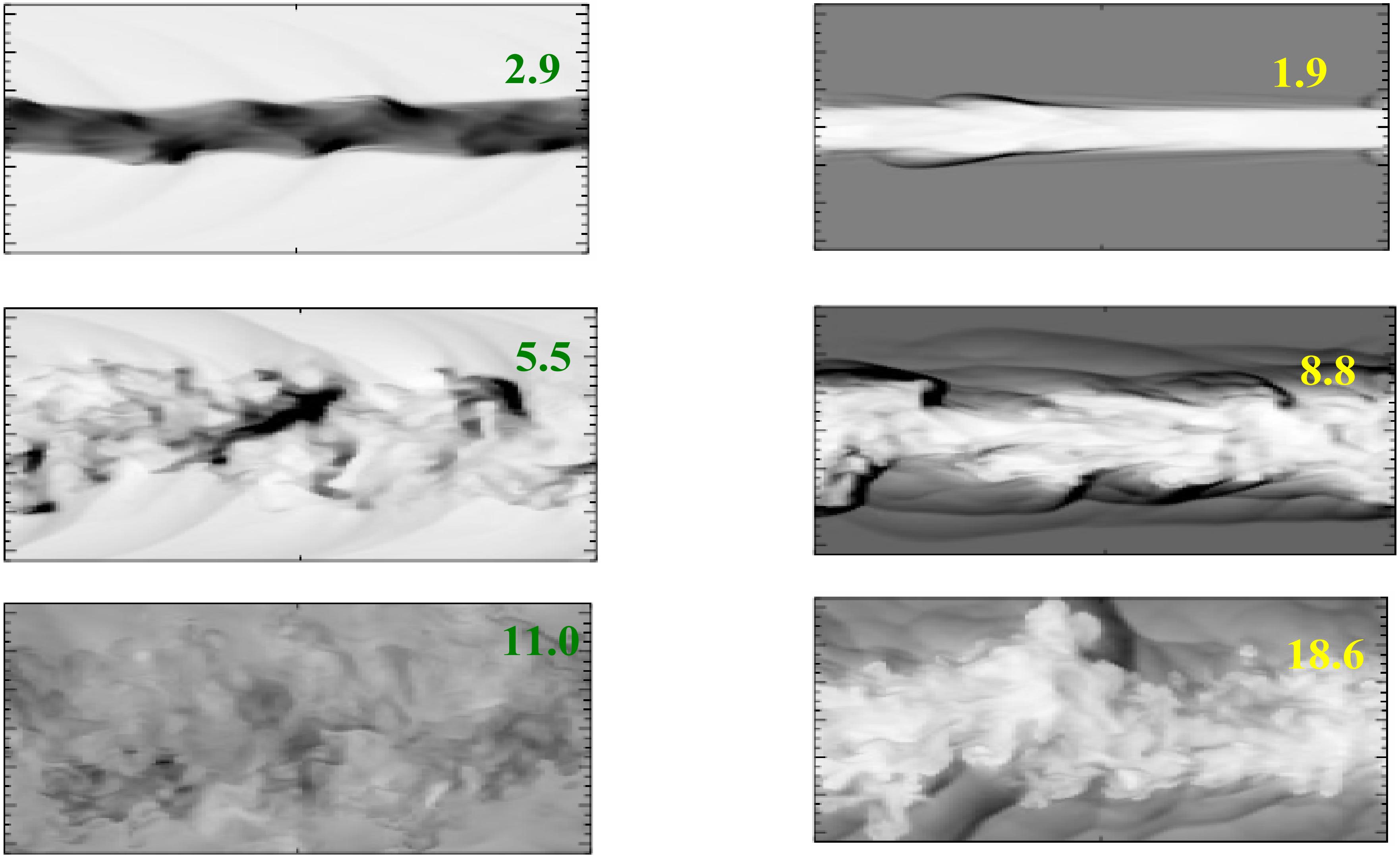
$\nu = 0.1$ $\nu = 10$



# Evolution of a 3-D supersonic jet

$M = 10$

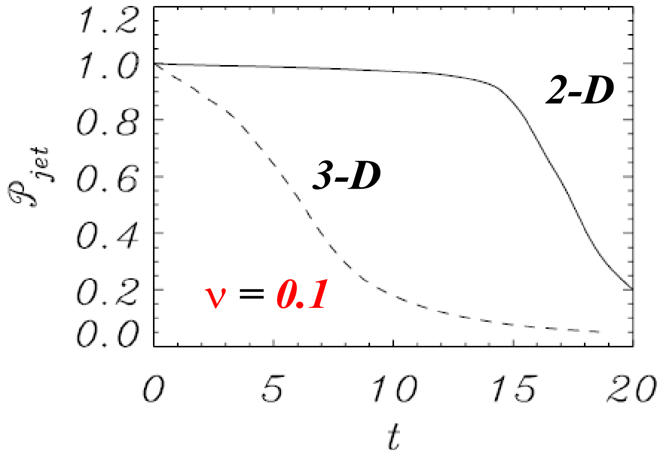
$\nu = 10$ $\nu = 0.1$

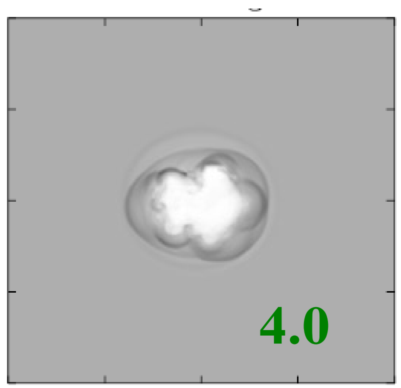


# 2-D vs 3-D $M = 10$

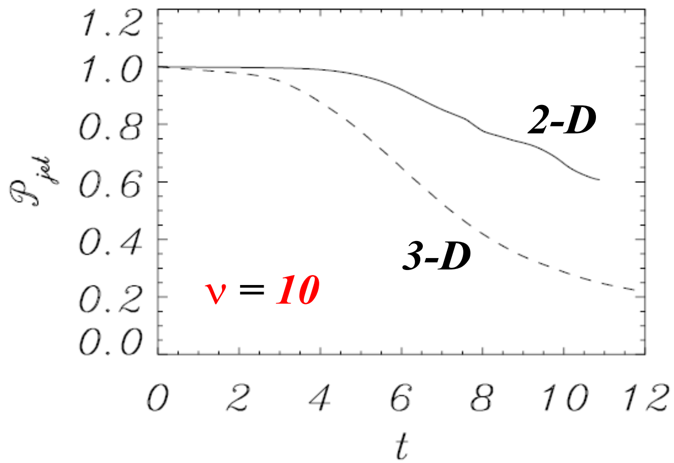
## Momentum $\mathcal{P}_{jet}$

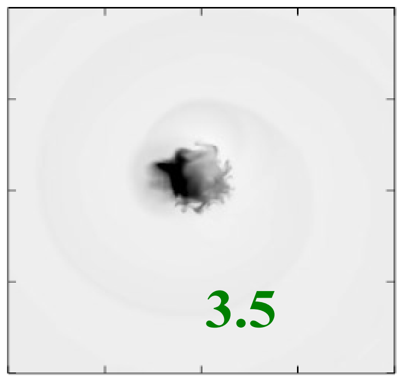
$\nu = 0.1$



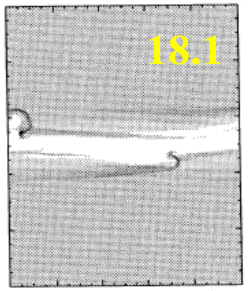

$\nu = 10$

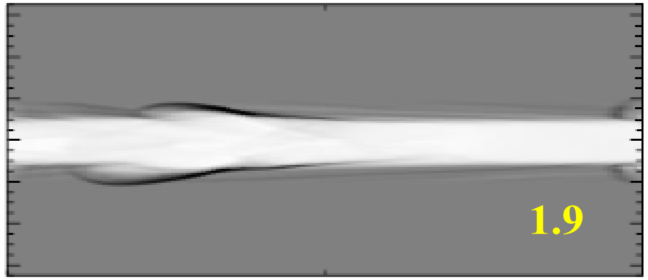


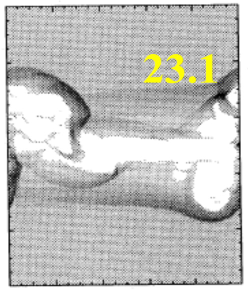



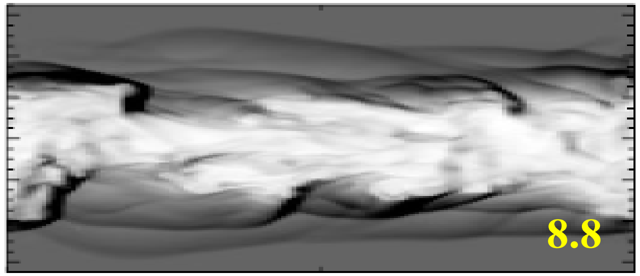
# 3-D vs 2-D $M = 10, \nu = 0.1$

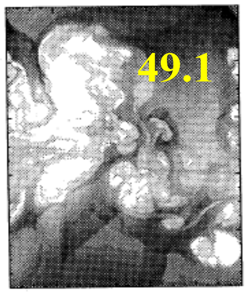
**2-D/s** **3-D**





Similar asympt. configurations but
***3-D jets evolve much faster than 2-D jets***

Shocks are ***weaker*** and the post shock material is ***cooler***

Mixing and momentum deposition ***are enhanced***

Large scale configur.: The ***helical mode*** prevails at the beginning

***but***

Rapid development of ***small scale structures***:
i) ***unstable n > 1*** modes (first)
ii) ***cascade*** to small scales eddies via ***non linear turbulent processes*** (later)

# ***KHI*** **vs** ***Astrophysical Jets***

***Do jet survive against the KHI ????***

More '*astrophysical*' ingredients must be considered:

- ***Radiative losses***

- ***Magnetic field B***

(- Relativistic regime)

# *Radiative Losses: KHI and thermal instabilities*

Relevant if $\tau = t_{cool}/t_{dyn} \leq 1$, $t_{cool} \approx P/\mathcal{L}$

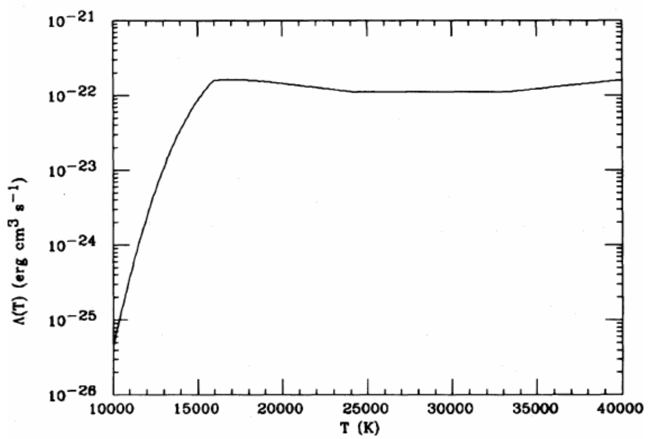
$$\mathcal{L} = n^2 \Lambda(T)$$

We can express $\Lambda(T) \propto T^{\alpha}$ where the value of $\alpha$ depends on $T$

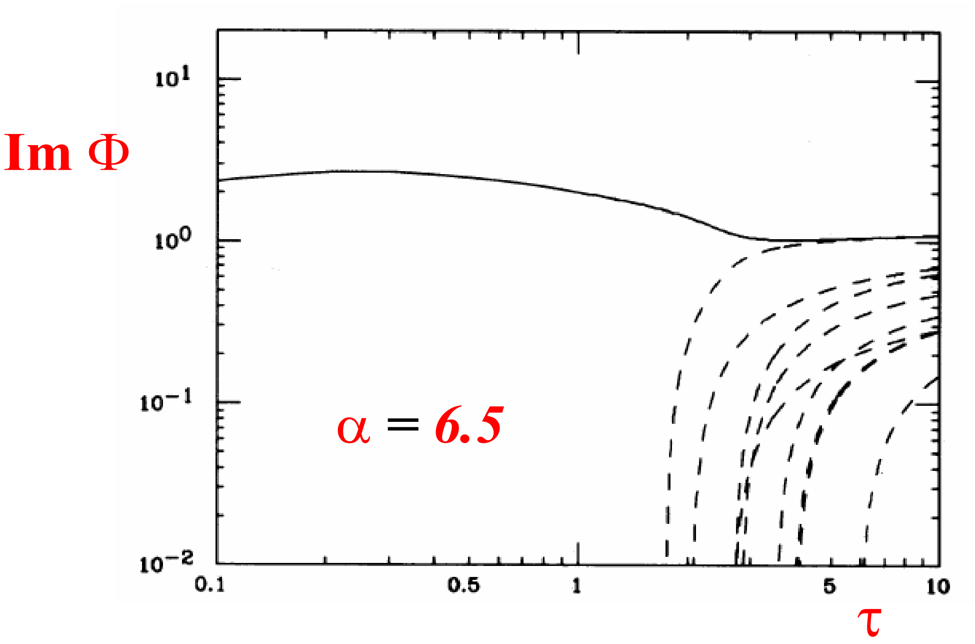
For an ***optically thin plasma***

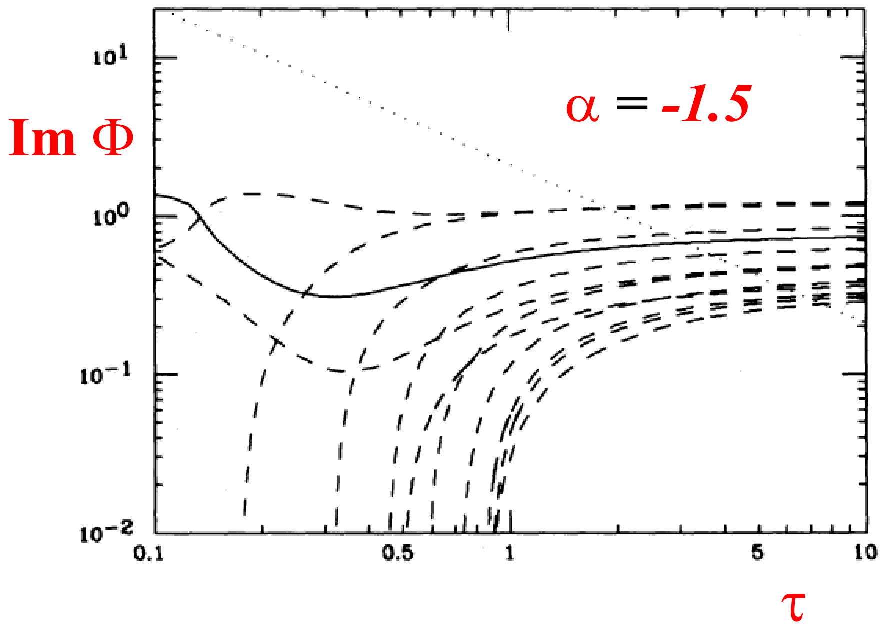
$\alpha = 6.5 \qquad T \leq 15.000\ K$

$\alpha = -1.5 \qquad 15.000\ K \leq T \leq 25.000\ K$



## $M = 10$ , $ka = 5$




***Ordinary modes*** slightly affected

***Reflected modes*** tend to be stabilized

Possible onset of the ***classical thermal instab***.

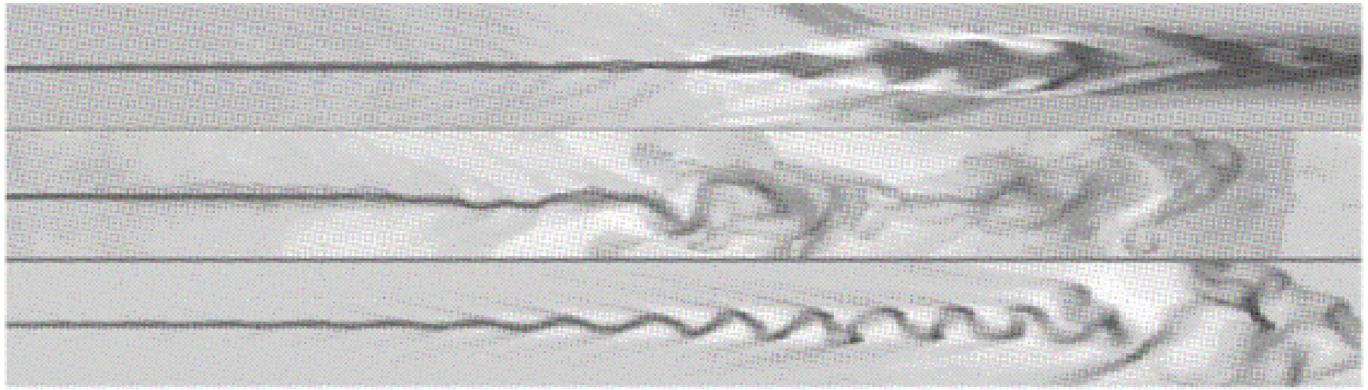
Different values of $\alpha$ ?

# ***Non linear evolution***: Which $\Lambda(T)$ ?

*Adiab.*

*a*=*0.5*

*a*=*2.5*



## ***Simulations of 3-D radiative jet:***

$\Lambda$: *Lines emission* (reson., forb.) from ***9*** elem. (sol. ab.), included the ionization level of ***H***

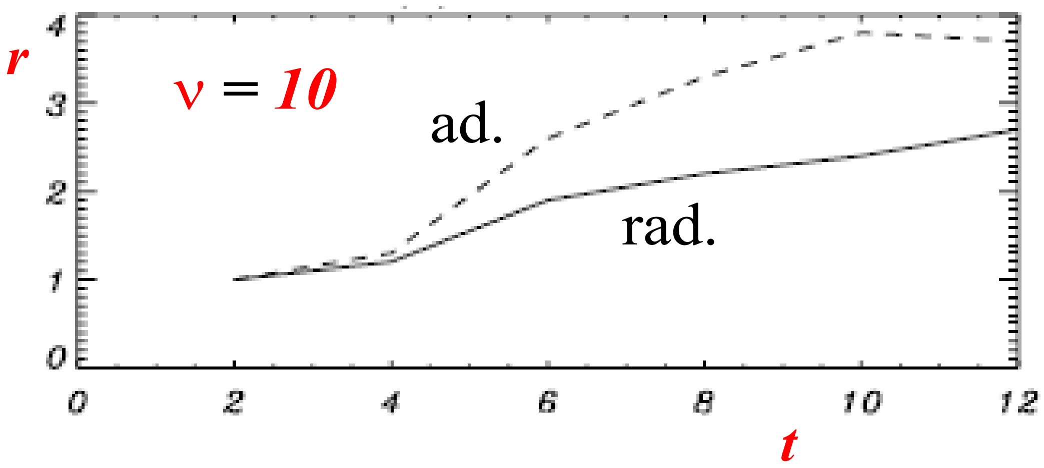
Parameters: $M = 10$, $T_o = 10.000$ K, $n_o = 100$ cm$^{-3}$

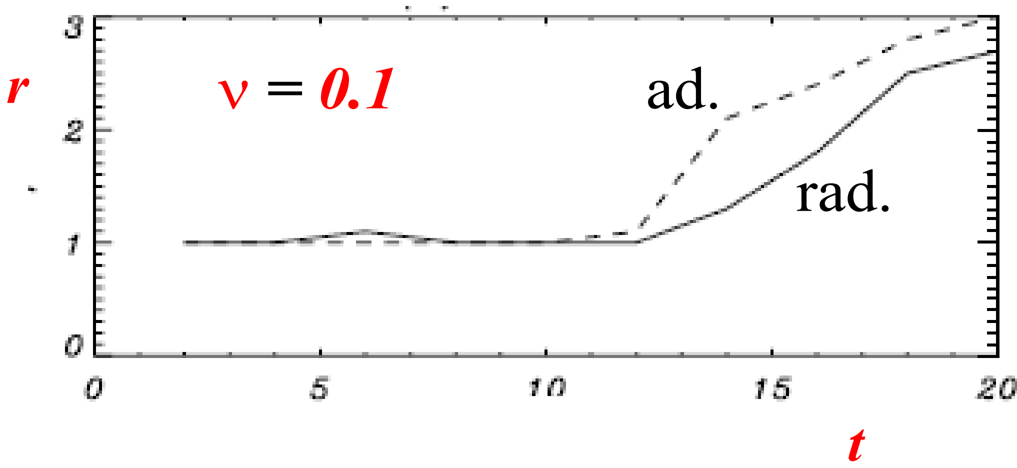
Radiation *delays* the effect of ***KHI,*** mainly in *dense jets*

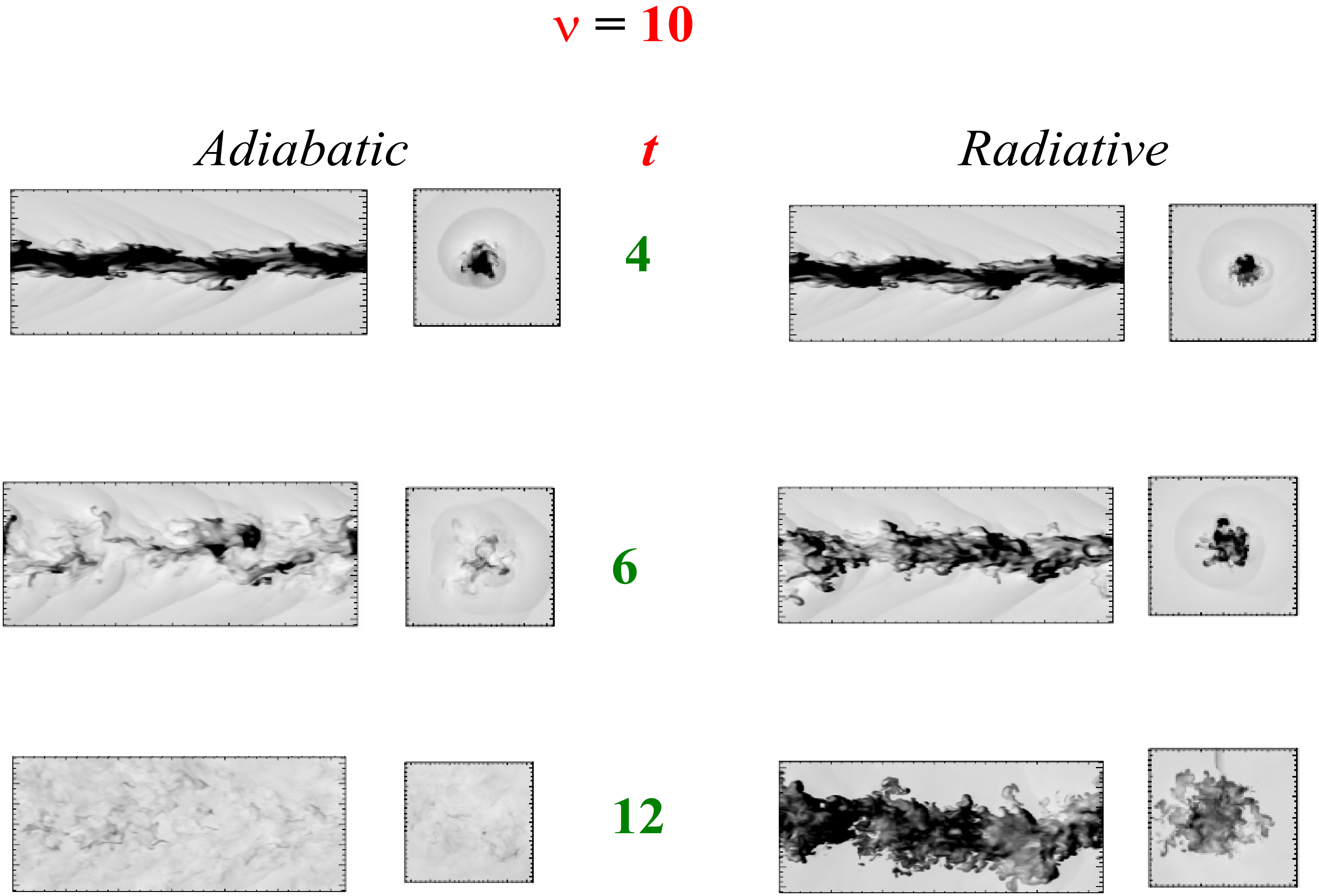
***Lower temperature*** in the post-shock region

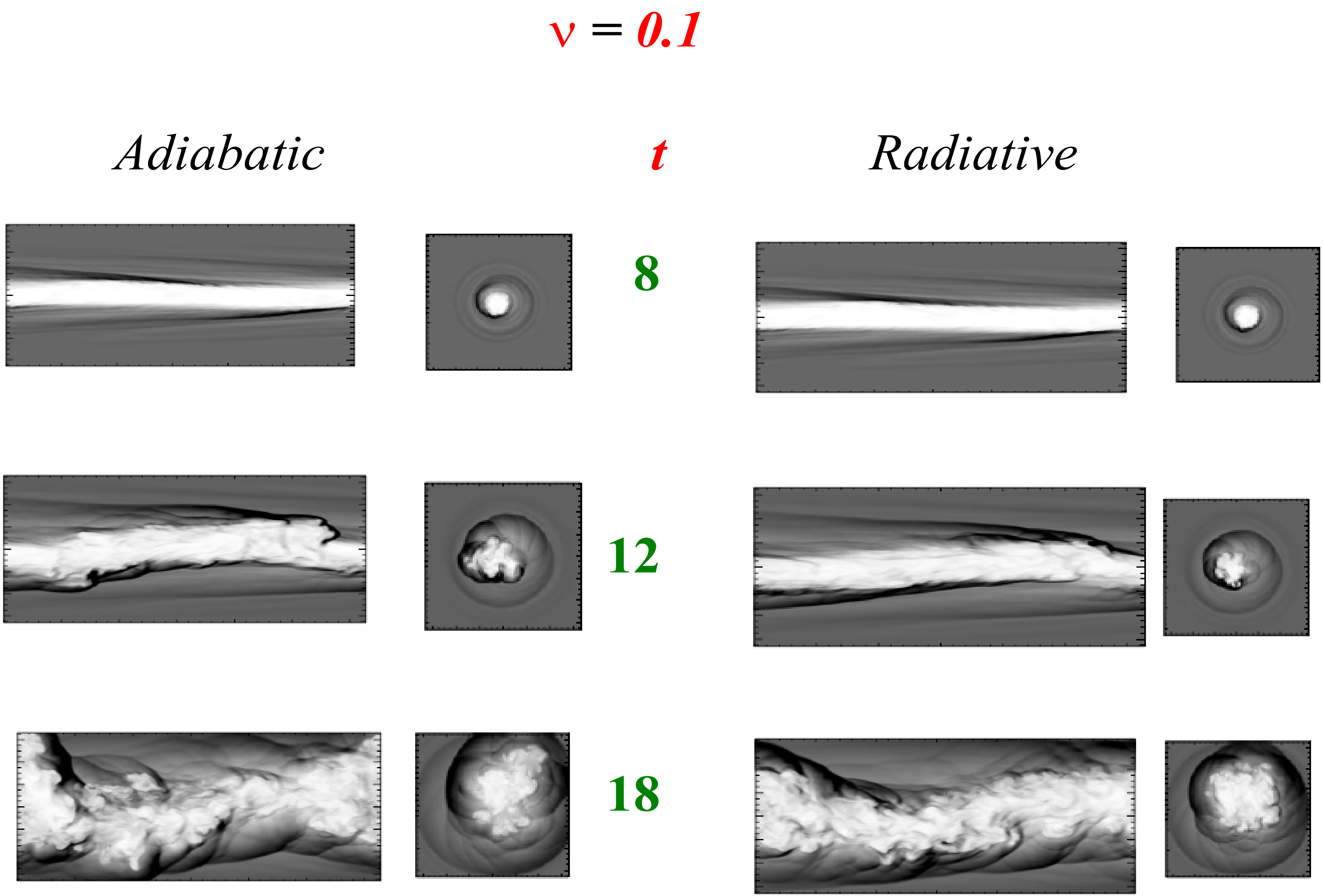
- ➔ *Smaller **entrainment** and **momentum** deposition*
- ➔ ***Disruption of jet delayed***
- ➔ ***No thermal instability***

***Evolution of the jet radius***



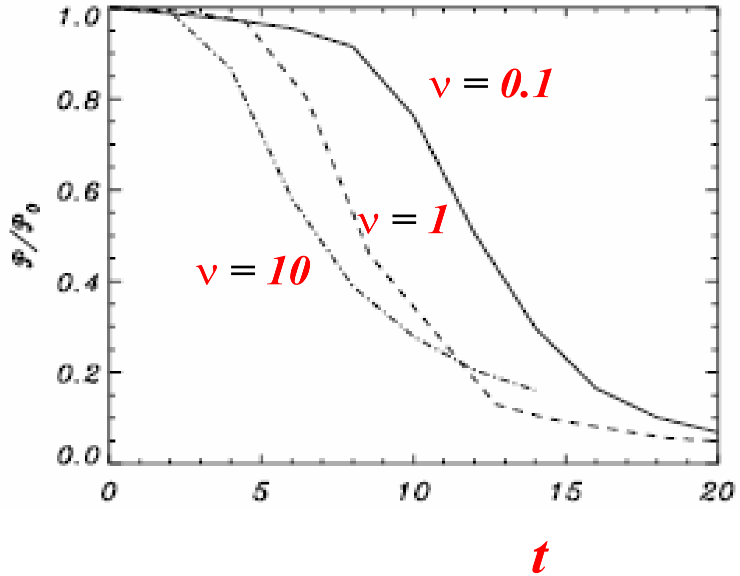




ν = 10
Adiabatic
t
Radiative
4
6
12


ν = 0.1
Adiabatic
t
Radiative
8
12
18

# *Interaction with the environment*

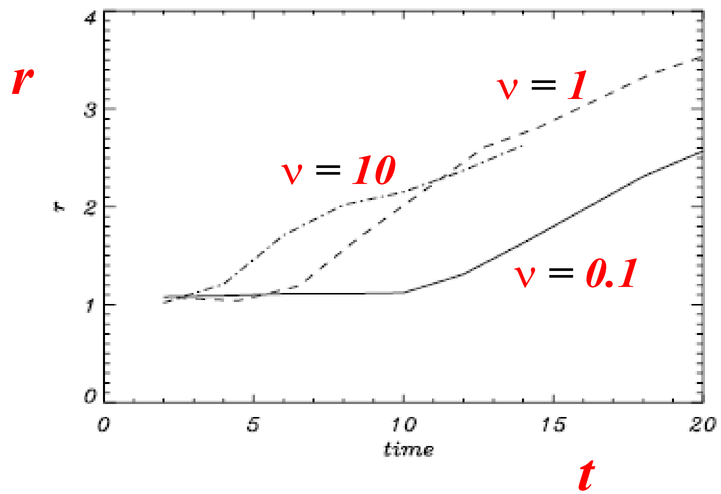
- The momentum is transferred to the environm. whithin $t \sim 20$




- The jet radius increases linearly

$v_{exp} \sim 0.15\ s$

$d_{\perp} = v_{exp}\ t\ = 4.5 \times 10^{17}\ s_6\ t_5$ cm



# *Linear KHI and magnetized jets*: *B* || v

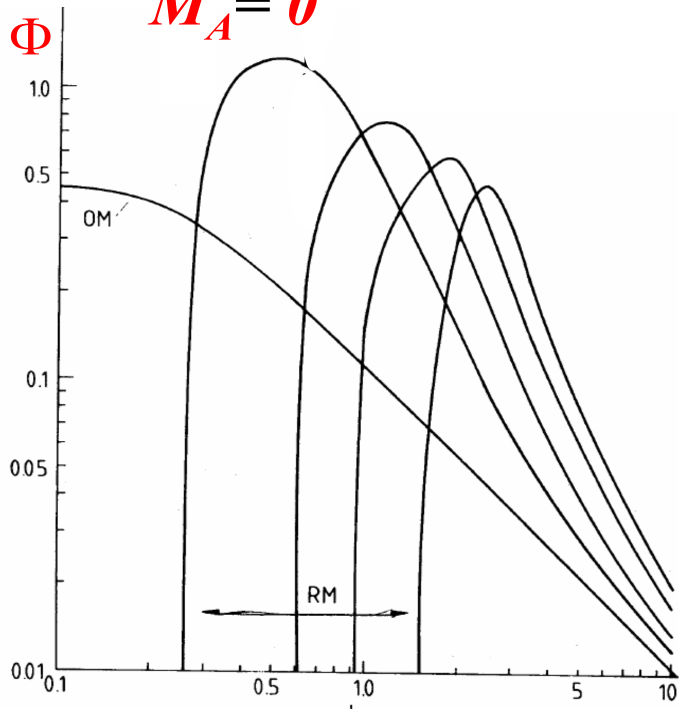
Unstable modes damped:

*RM*: shifted to *ka* >> *1*

*OM*: unstable for $M_A < 0.5 - 1$

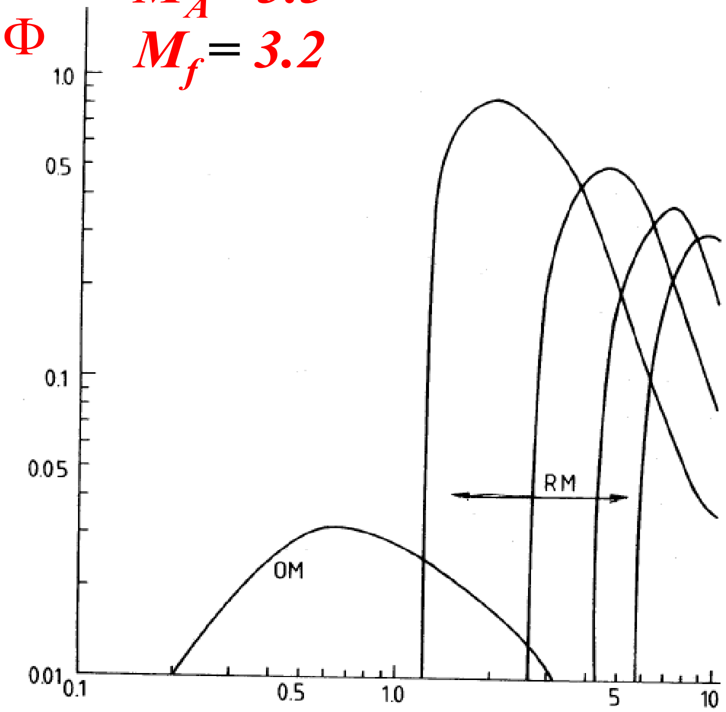
$M=10$, $n=0$

Im Φ $M_A = 0$




*ka*

Im Φ $M_A = 3.3$ $M_f = 3.2$




*ka*

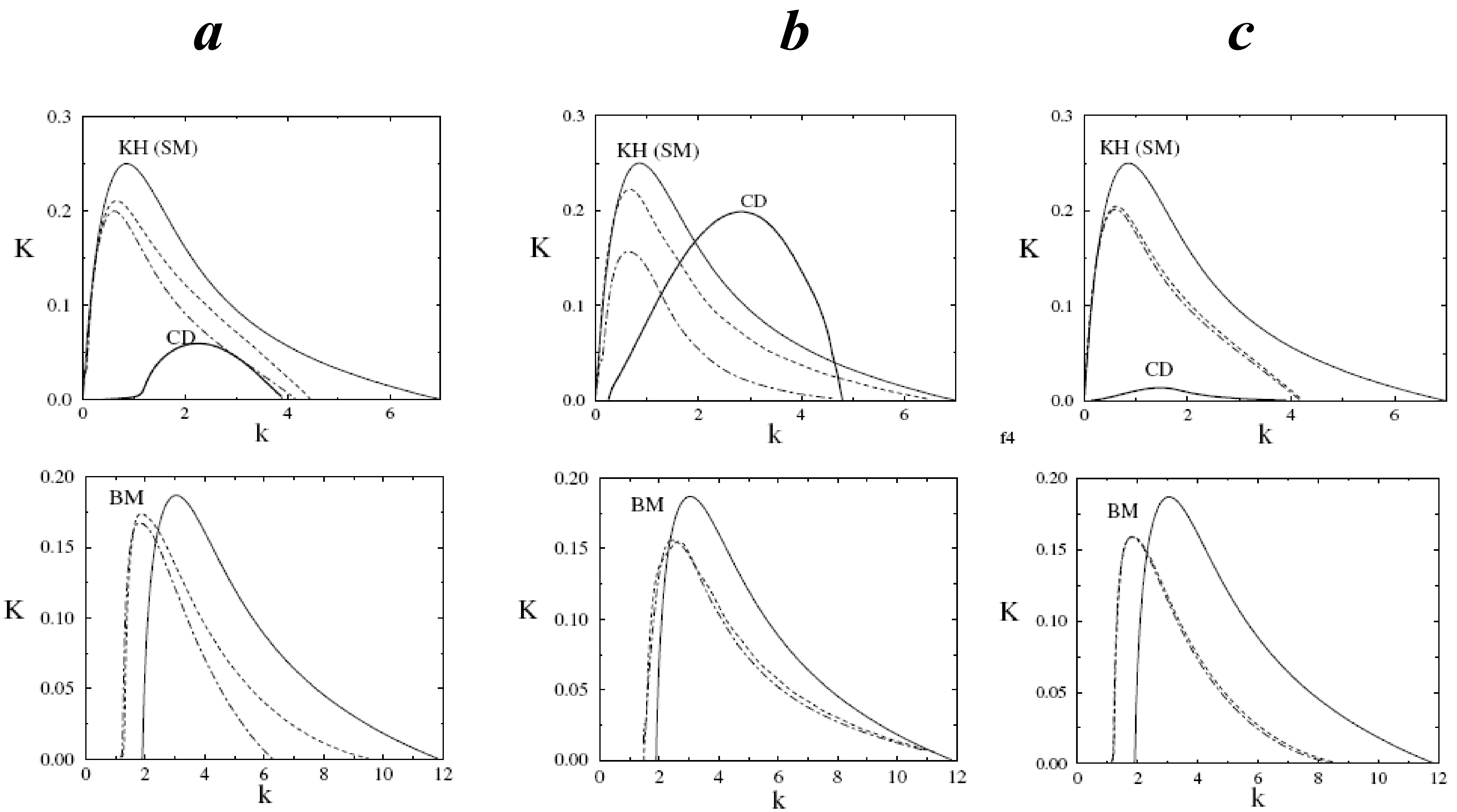
# Linear KHI and magnetized jets: $B_\phi \neq 0 \rightarrow$ CDI

**Which equilibrium for B ?**

a - $rB_z / B_\phi = cst$

b - $B_z = cst$

c - $B_\phi = 0,\ r > a$




$M_f = 3$

$V_A/s = 1$

$M = M_A = 4.2$

$|n| = 1$

- Splitting of modes with $n = 1$ and $n = -1$

- The *KHI* tends to be damped for $B_\phi \neq 0$

- This trend seems to be quite independent on the initial equilibrium

- The damping is stronger for $V_A / s >> 1$

- The onset of the *CDI* critically depends on the equilibrium

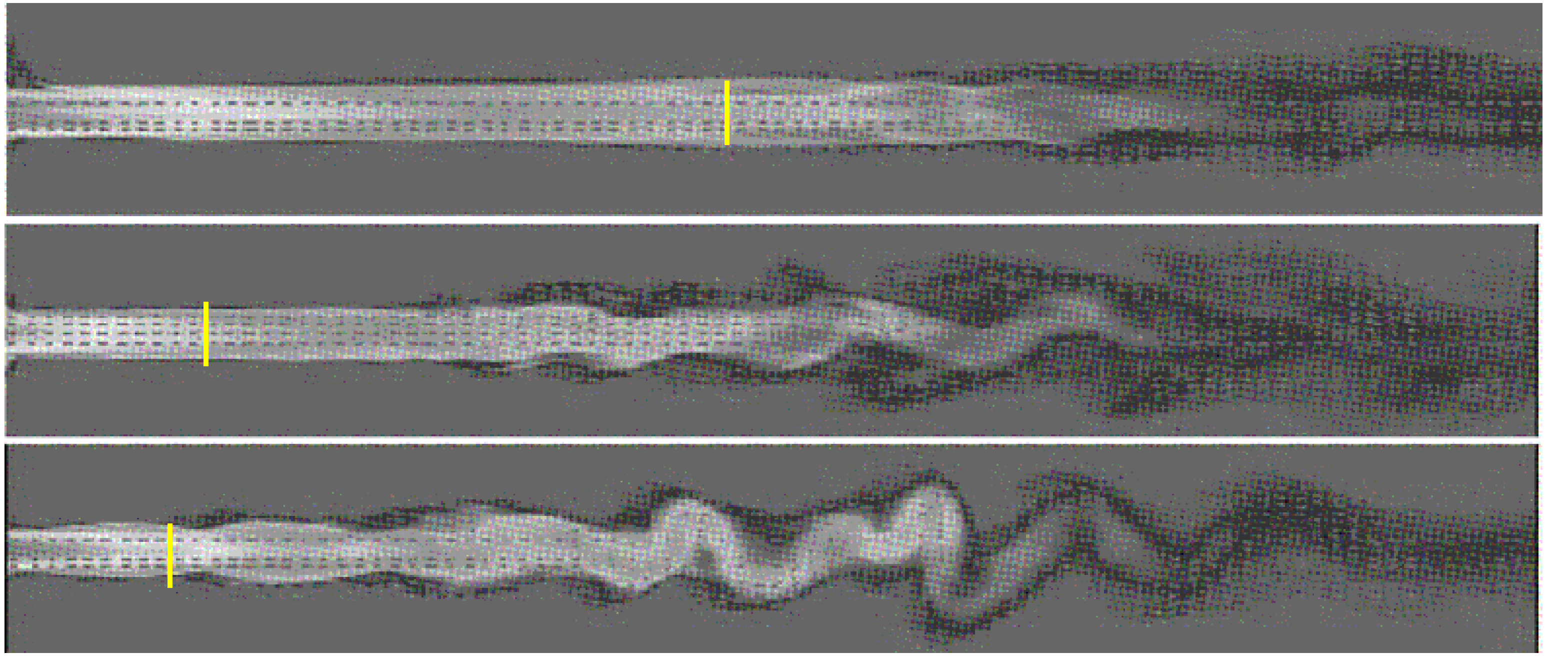
- Sometimes the *CDI* can prevail over the *KHI*

-

# Nonlinear evolution of magnetized KHI

***Spatial analysis of a transalfvenic, slowly expanding jet***

$M \sim 1 - 1.8$,  $M_f \sim 0.6$,  $M_A \sim 0.7 - 0.9$



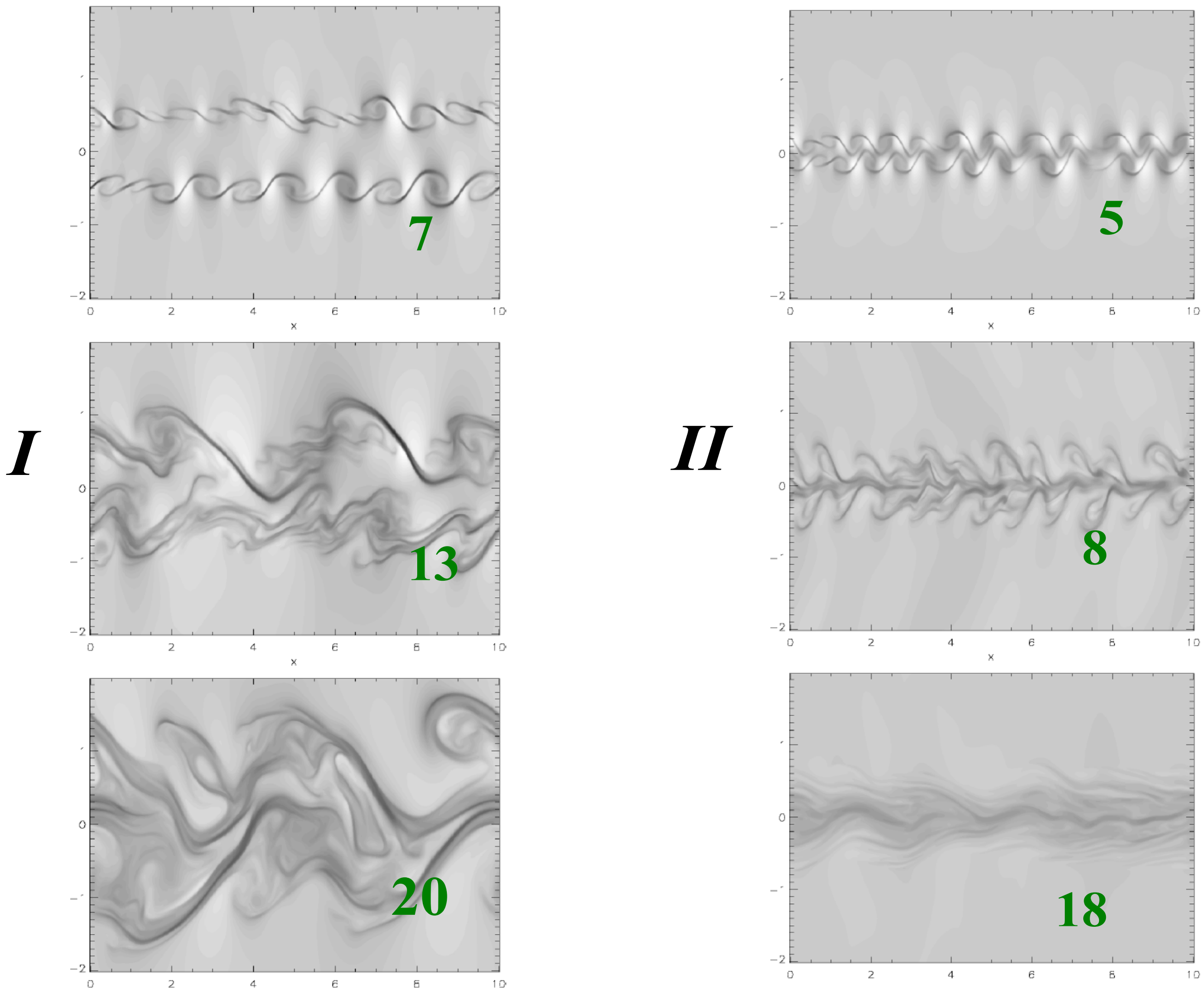
$B_\phi = 0$

"

$B_\phi \neq 0$

# *Non linear evolution*: $B_\phi = 0$, *2-D*

$M \sim M_f \sim 1,\ M_A = 7,\ V_A / s \ll 1$




The slab evolution strongly depends on:

$a / d_{tr}$

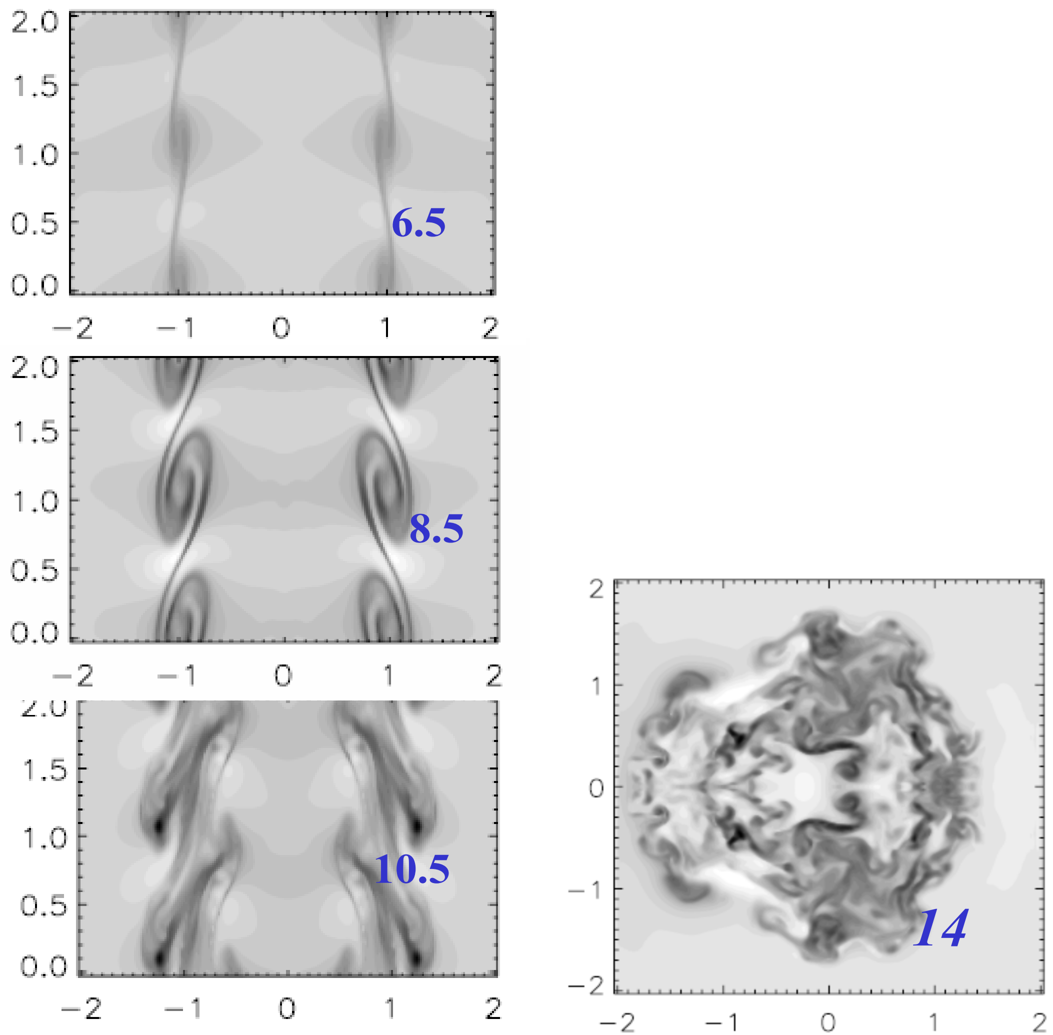
*I* - $a / d_{tr} \gg 1$

*II* - $a / d_{tr} \sim 1$

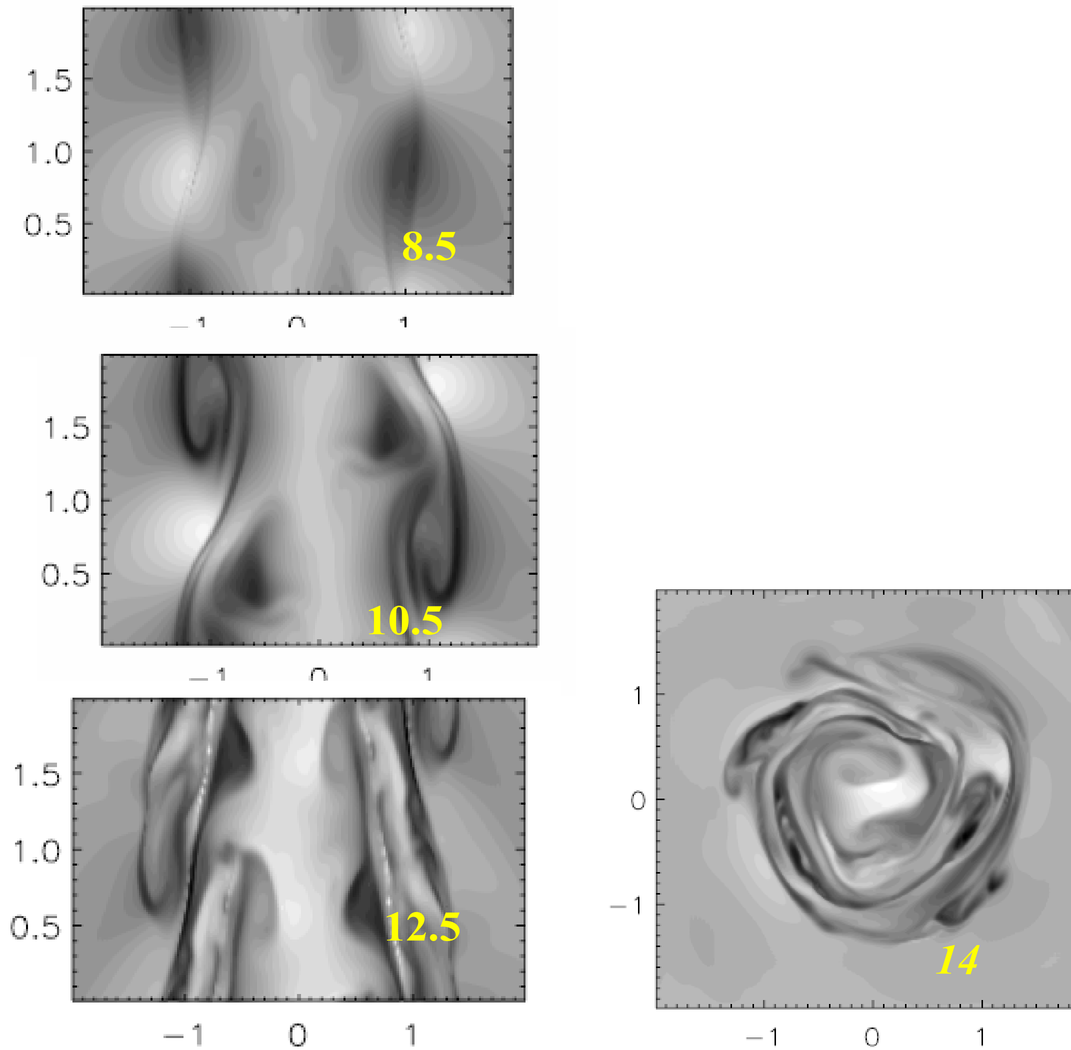
# *Non linear evolution: 3 - D*

$M = 1.3, \; M_f = 1.24, \; M_A = 0.6, \; V_A / s \sim 0.5, \; B_z = cst$

$B_\phi = 0$




$B_\phi \neq 0$ (CDI)



# *Non linear evolution 3-D* (*ctd*)

***3-D*** jet with ***longitudinal magnetic*** field ***evolves more rapidly***

A ***toroidal magnetic field*** may have a ***stabilizing effect*** on the development of turbulence

Apparently this is due to the onset of the ***CDI*** that ***increasees*** $B_{\phi}$ on the boundary

***hoop stress counteracts the KHI***

# Summary

***Do supersonic jets survive against KHI ?***

$$t_{cr} = 350\ a_{16} s_6^{-1} \quad \text{yrs}$$

***Radiation*** + ***magnetic fields*** ~ ***Yes***

***KHI*** plays a main role in the interaction between the jet and the environment:

***Momentum, energy deposition***
***Entrainment, Mixing***
***Shocks, heating of the gas***
**(** ***periodic knots ?*** **)**

………………………………………

***What next ?*** ***3-D*** supersonic jets (***body modes***)

+ ***helical fields*** (equilibrium ***B***)

+ ***radiative losses*** (cooling function)

***3-D*** Shock evolution

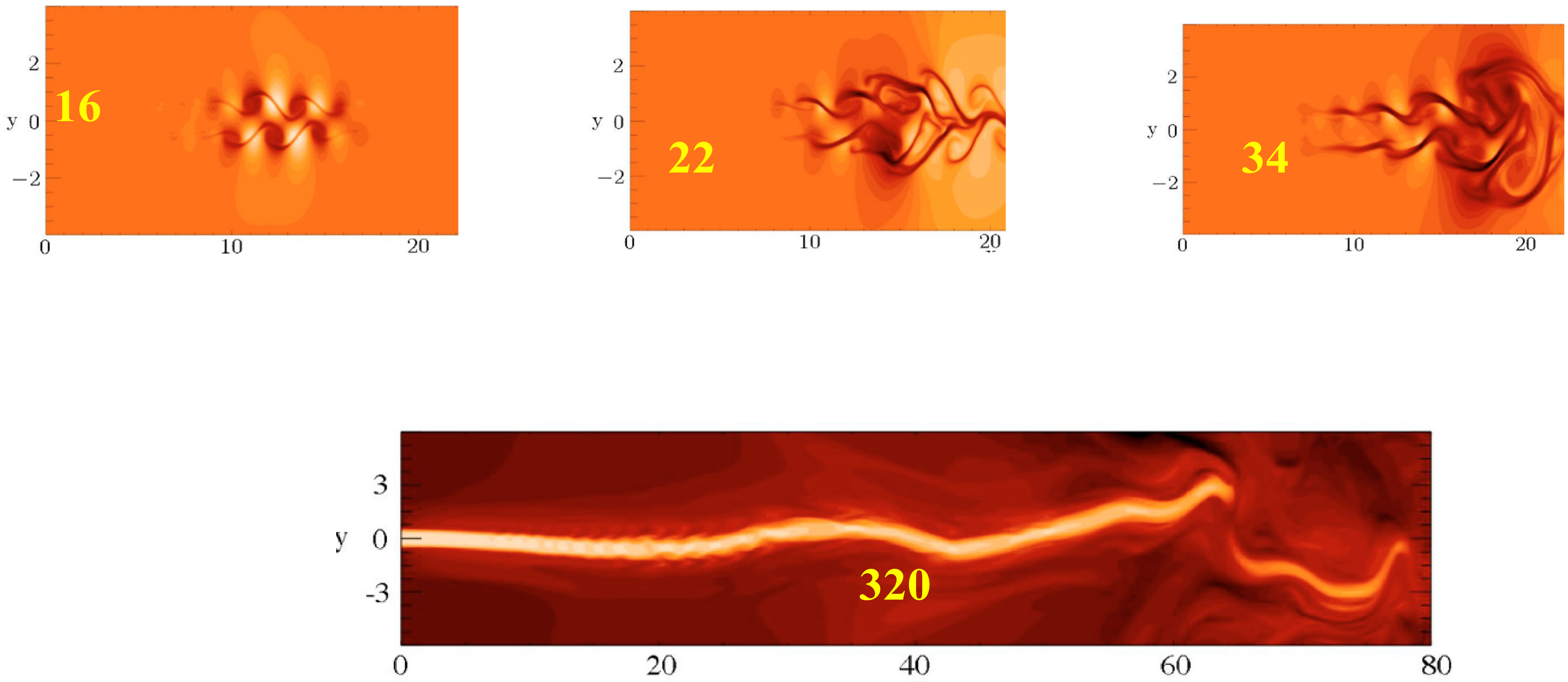
Transition layer, resistivity

***Warnings***: ***2 - D*** vs ***3 - D*** simulations

***Temporal*** vs ***spatial*** simulations

# *Non linear spatial evolution*: $B_\phi = 0$, *2-D*

$M = 1,\ M_f = 0.99,\ V_A / s = 0.14$



***References***:          ...............................

***Hardee et al***. (*Al., Usa*)
***Appl, Baty, Keppens et al***. (*Heidelberg, Strasbourg, Leuven*)
***Bodo, Massaglia et al***. *(Torino)*

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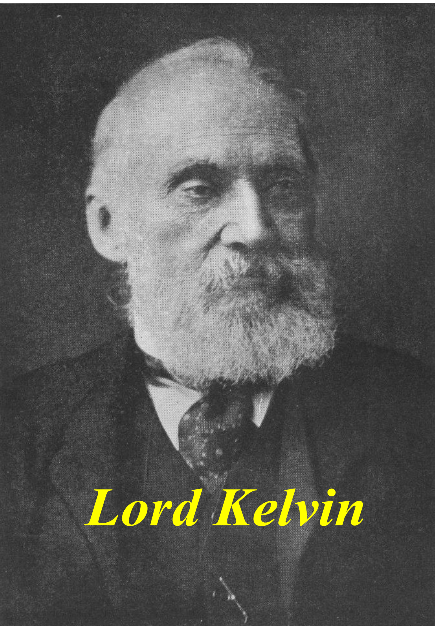


***Thank You***

***very much !***