Extending analytical MHD jet formation models with a finite outer disk radius

Matthias Stute[1], Kanaris Tsinganos[1], Nektarios Vlahakis[1], Titos Matsakos[2], Jose Gracia[3]

[1]IASA and Section of Astrophysics, Astronomy and Mechanics, Department of Physics, University of Athens, Panepistimiopolis, 157 84 Zografos, Athens, Greece

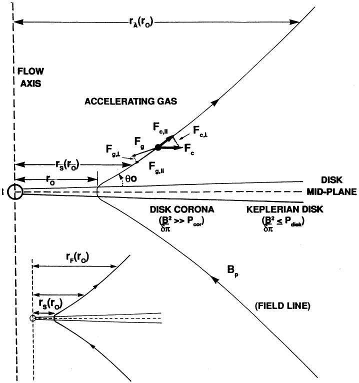
[2]Dipartimento di Fisica Generale, Università degli Studi di Torino, via Pietro Giuria 1, 10125 Torino, Italy

[3]School of Cosmic Physics, Dublin Institute of Advanced Studies, 31 Fitzwilliam Place, Dublin 2, Ireland

July 8, 2008

Outline

- Motivation
- Numerical models
- Stability and structure of solutions
- Application to observations
- Discussion



Pelletier & Pudritz (1992)

- the magnetic field lines are like rigid wires controlling the flow like beads
- acting on a gas parcel are gravity (of the central star) F_g and centrifugal forces F_c due to Keplerian rotation $\Omega_{\rm K}$ accelerating the flow
- collimation by the toroidal component of the magnetic field (magnetic hoop stress)

Analytical MHD jet formation models in a nutshell

MHD equations:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) &= 0\,, \\
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} + \frac{1}{\rho} \vec{B} \times (\nabla \times \vec{B}) + \frac{1}{\rho} \nabla p &= -\nabla \Phi\,, \\
\frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p + \Gamma p \nabla \cdot \vec{v} &= \Lambda\,, \\
\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{v} \times \vec{B}) &= 0\,, \\
\nabla \cdot \vec{B} &= 0\,.
\end{aligned}
$$

Analytical MHD jet formation models in a nutshell

Assumptions:

I Stationarity $\Longrightarrow \frac{\partial}{\partial t} = 0$

II Axisymmetry $\Longrightarrow \frac{\partial}{\partial \phi} = 0$

$\Longrightarrow$ Existence of Invariants:

$$\Psi_A(A) = \frac{\rho\, v_p}{B_p}\,, \quad \Omega(A) = \frac{1}{R}\left(v_\phi - \frac{\Psi_A\, B_\phi}{\rho}\right)\,, \quad L(A) = R\left(v_\phi - \frac{B_\phi}{\Psi_A}\right)\,.$$

If $\Lambda = 0$,

$$E(A) = \frac{v^2}{2} + \frac{\Gamma}{\Gamma - 1}\,\frac{p}{\rho} + \Phi - \Omega\, R\,\frac{B_\phi}{\Psi_A}\,, \qquad Q(A) = \frac{p}{\rho^\Gamma}\,.$$

III Radial self-similarity $\Longrightarrow \mathcal{Q} = \mathcal{Q}_0\left(\frac{R}{R_0}\right)^\alpha \mathcal{F}\left(\frac{z}{R}\right)$

Motivation

Known problems:

- Causality

$\Longrightarrow$ no feedback is allowed
$\Longrightarrow$ flow must go through all three critical points

$$\text{Alfven waves} \qquad v_{\mathrm{A}} = \frac{B}{\sqrt{\mu_0\, \rho}}$$

$$\text{slow- and fast-magnetosonic waves} \qquad v_{\mathrm{SM,FM}}^2 = \frac{1}{2}\left(v_{\mathrm{A}}^2 + c_{\mathrm{s}}^2 \mp \sqrt{(v_{\mathrm{A}}^2 + c_{\mathrm{s}}^2)^2 - 4\, v_{\mathrm{A}}^2\, c_{\mathrm{s}}^2\, \cos^2\theta}\right)$$

Condition not fulfilled in e.g. Blandford & Payne (1982).

Vlahakis et al. (2000) showed how to construct solutions which crosses all critical points.

Motivation

Known problems:

- Singularity close to jet axis

$$\mathcal{Q} = \mathcal{Q}_0 \left(\frac{R}{R_0}\right)^{\alpha} \mathcal{F}\left(\frac{z}{R}\right)$$

$$R \to 0 \Longrightarrow \mathcal{Q} \to \infty$$

Numerical simulations are needed to extend the analytical solutions towards the axis, as e.g. Gracia et al. (2006) and Matsakos et al. (2008).

Motivation

Known problems:

Self-similarity

$\Longrightarrow$ there is no preferred scale, i.e. jet extends formally to infinite radii

$\Longrightarrow$ jet driving accretion disk, however, has finite outer radius

Numerical simulations are needed to study the effects of imposing an outer radius on the topology, structure and variability of a radially self-similar analytical MHD solution.

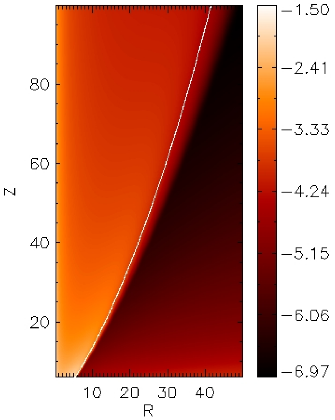
Our approach

Parallel use of analytical and numerical methods:

$\Longrightarrow$ numerical simulations to extend the analytical solution
$\Longrightarrow$ analytical methods to interpret and understand the outcome of the simulations

- Numerical simulations with PLUTO (Mignone et al. 2007)
- Initial conditions closely based on the well-known analytical disk outflow (ADO) solution of Vlahakis et al. (2000) which crosses all critical points
- Modification of the ADO solution at small radii as in Gracia et al. (2006) and Matsakos et al. (2008)
- Truncation of the ADO solution at different radii

Initial conditions



- Domain divided into two regions, an inner region and an outer region
- Inner region (up to a truncation field line) is fully determined by analytical solution of Vlahakis et al. (2000)
- How to initialize outer region?

Initial conditions

Test simulations showed that either the toroidal magnetic field component B_ϕ should be very small at the outer radial boundary or that all quantities should be modified self-consistently in order to maintain an equilibrium in the external region.

$$\text{Let } \left[\rho\,(R,Z)\,,p\,(R,Z)\,,\vec{v}\,(R,Z)\,,\vec{B}\,(R,Z)\right] \text{ be a solution of the MHD equations}$$

$$\Downarrow$$

$$\left[\rho\,'\,(R\,',Z\,')\,,p\,'\,(R\,',Z\,')\,,\vec{v}\,'\,(R\,',Z\,')\,,\vec{B}\,'\,(R\,',Z\,')\right] \text{ is also a solution of the same set of equations, if}$$

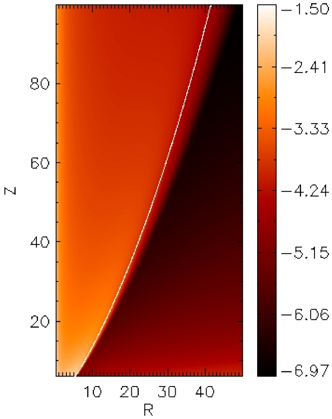
Scalings

$$R\,' = \lambda_1\,R\,, \qquad Z\,' = \lambda_1\,Z\,, \qquad \vec{B}\,' = \lambda_2\,\vec{B}\,, \qquad \vec{v}\,' = \sqrt{\frac{\lambda_3}{\lambda_1}}\,\vec{v}\,,$$

$$\rho\,' = \frac{\lambda_1\,\lambda_2^2}{\lambda_3}\,\rho\,, \qquad p\,' = \lambda_2^2\,p\,, \qquad \mathcal{M}\,' = \lambda_3\,\mathcal{M}$$

Same central object $\Longrightarrow \mathcal{M}\,' = \mathcal{M} \Longrightarrow \lambda_3 = 1 \Longrightarrow \lambda_1,\ \lambda_2$ two free parameters

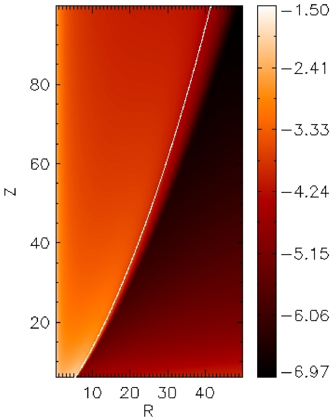
Initial conditions



- Domain divided into two regions, an inner region and an outer region
- Inner region (up to a truncation field line) is fully determined by analytical solution of Vlahakis et al. (2000)
- Scaled solution in outer region, matched with

$$
\begin{aligned}
\mathcal{Q} &= \mathcal{Q}_{\rm in}\ \exp[-(\alpha/\alpha_{\rm trunc})^2] \\
&\quad + \mathcal{Q}_{\rm out}\ \left(1 - \exp[-(\alpha/\alpha_{\rm trunc})^2]\right)
\end{aligned}
$$

Boundary conditions



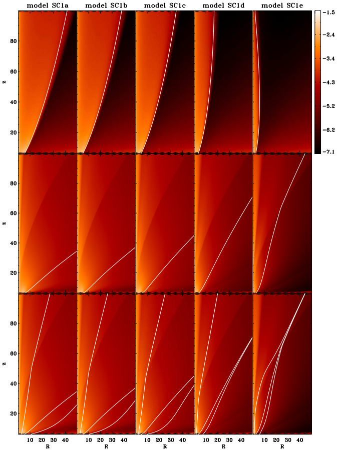
- Outflow conditions at outer radial and top z boundary
- Axisymmetry at inner radial boundary
- At the lower boundary, we keep the quantities *fixed* to their analytical values, however, making sure that the problem is not over-specified.

Parameter study

Models and parameters

Name	Description
SC1a	$\alpha_{\text{trunc}} = 0.4$ ($R_{\text{tr}} = 5.325\,R_0$), external analytical solution $\lambda_1 = 10^3$, $\lambda_2 = 10^{-3}$
SC1b	$\alpha_{\text{trunc}} = 0.2$ ($R_{\text{tr}} = 5.125\,R_0$), external analytical solution $\lambda_1 = 10^3$, $\lambda_2 = 10^{-3}$
SC1c	$\alpha_{\text{trunc}} = 0.1$ ($R_{\text{tr}} = 4.875\,R_0$), external analytical solution $\lambda_1 = 10^3$, $\lambda_2 = 10^{-3}$
SC1d	$\alpha_{\text{trunc}} = 0.01$ ($R_{\text{tr}} = 3.625\,R_0$), external analytical solution $\lambda_1 = 10^3$, $\lambda_2 = 10^{-3}$
SC1e	$\alpha_{\text{trunc}} = 0.001$ ($R_{\text{tr}} = 2.625\,R_0$), external analytical solution $\lambda_1 = 10^3$, $\lambda_2 = 10^{-3}$
SC2	$\alpha_{\text{trunc}} = 0.4$, external analytical solution $\lambda_1 = 100$, $\lambda_2 = 0.1$
SC3	same as model SC2, but solutions are swapped
SC4	$\alpha_{\text{trunc}} = 0.4$, external analytical solution $\lambda_1 = 1$, $\lambda_2 = 0.1$
SC5	same as model SC4, but solutions are swapped

Structure of the final models

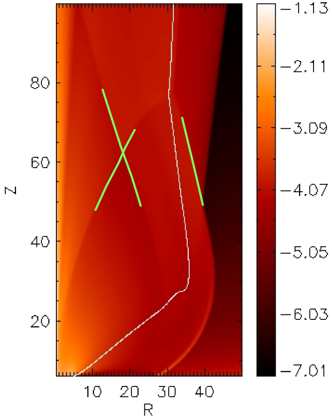


Models with scaled solution outside

Movie I (gif)

Movie I (mpg)

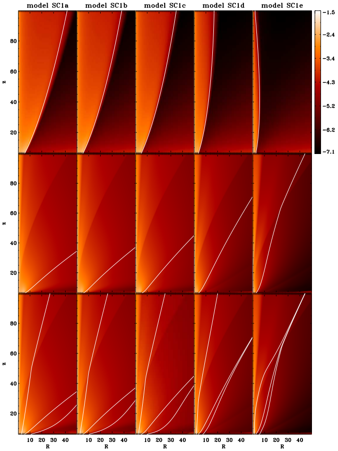
Structure of the final models



Shocks

- slow- and fast-magnetosonic waves which transport downstream the effect of the boundary condition at the base, namely the truncation of the solution (present in all models, but not in ADO as expected)
- fast-magnetosonic separatrix surface (FMSS) which shields the flow from the modification close to the axis (present in all models, including ADO as presented by Gracia et al. 2006, Matsakos et al. 2008)
- field line are bent by shocks

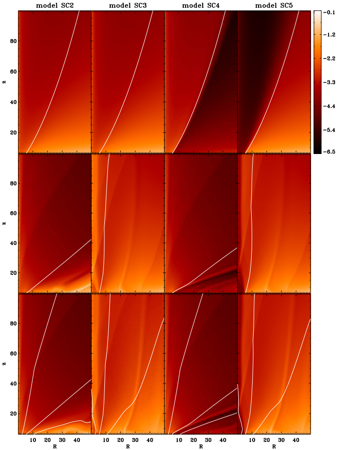
Structure of the final models



Models with scaled solution outside

- At the beginning, a shock front starting at the jet base runs across the jet, bending its outer surface and forming a dent which then travels out- and upwards
- a new smooth jet surface develops, sometimes with a larger radius than the initial one. This configuration is stable for several t_0 in all models.
- opening angles of about 40°–50° (emission is more collimated than density)

Structure of the final models

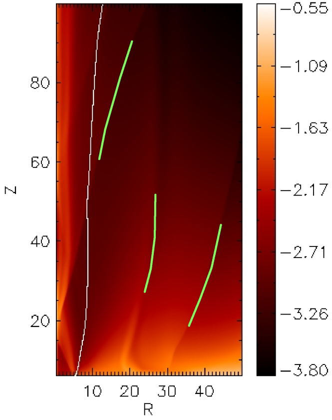


Models with scaled solution inside

Movie I (gif)

Movie I (mpg)

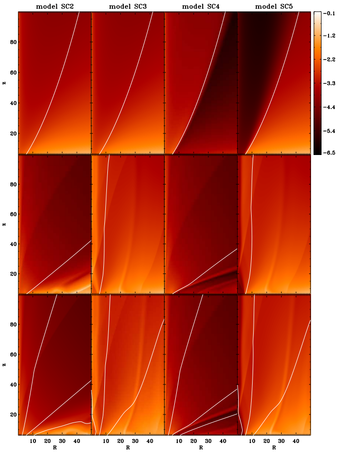
Structure of the final models



Shocks

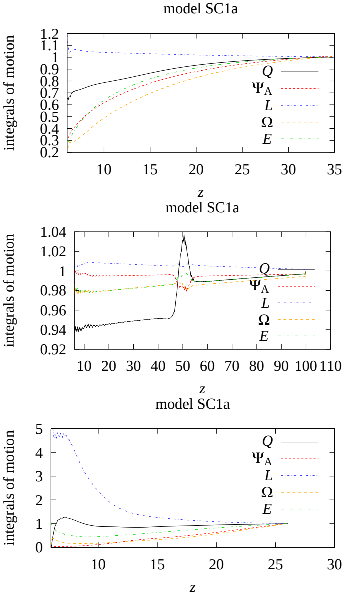
- same shock structure: FMSS, slow- and fast-magnetosonic waves
- slow-magnetosonic wave is locked to point where slow-magnetosonic critical surface crosses the lower boundary and develops a standing and steepening shock inside the domain
- fast-magnetosonic wave leaves domain

Structure of the final models



Models with scaled solution inside

- models show collimation due to exterior thermal and magnetic pressure
- opening angles of around 5°



- from assumptions of axisymmetry and stationarity directly follows existence of invariants
  $\Longrightarrow$ good test for simulation based on analytical methods
- integrals of motion converge well along truncation field line, an inner field line and an outer field line
  $\Longrightarrow$ models reach steady state in inner and outer region
- peaks in integrals along inner field line due to crossing of FMSS

Application to observations

Dimensions and units

- conversion of dimensionless quantities used by PLUTO to physical units necessary
- three normalization constants needed: jet velocity v_0, jet density ρ_0, mass of central object $\mathcal{M}$

$$
\begin{aligned}
R_0 &= \frac{\mathcal{G}\,\mathcal{M}}{4\,v_0^2} = 216.22\,\mathrm{AU}\left(\frac{v_0}{\mathrm{km\,s^{-1}}}\right)^{-2}\left(\frac{\mathcal{M}}{\mathrm{M_\odot}}\right) \\
B_0 &= \sqrt{4\,\pi\,\rho_0\,v_0^2} = 11.21\,\mu\mathrm{G}\left(\frac{\rho_0}{10^{-21}\,\mathrm{g\,cm^{-3}}}\right)^{1/2}\left(\frac{v_0}{\mathrm{km\,s^{-1}}}\right) \\
p_0 &= \rho_0\,v_0^2 = 10^{-11}\,\mathrm{g\,cm^{-1}\,s^{-2}}\left(\frac{\rho_0}{10^{-21}\,\mathrm{g\,cm^{-3}}}\right)\left(\frac{v_0}{\mathrm{km\,s^{-1}}}\right)^2
\end{aligned}
$$

from simulations: $\rho \approx 3.16 \times 10^{-5}\,\rho_0$, $p \approx 10^{-3}\,p_0$, $v_z \approx 100\,v_0$ and $B\phi \approx 0.1\,B_0$

(at the outer z boundary and $R = 25\,R_0$)

Application to observations

Calculations

- we use a set of tools described by Gracia et al. (2008) to produce synthetic observations from our simulations in different consecutive stages
  - approximation of the chemical composition of the plasma by locally solving a chemical network under the assumption of stationarity
  - calculation the statistical equilibrium of level populations for each ion of interest as a function of temperature and density and the emissivity for individual transitions of interest
  - integration along the line-of-sight and projection
  - convolution with a Gaussian point-spread-function (PSF)

$\Longrightarrow$ talk by J. Gracia on Thursday!

Application to observations

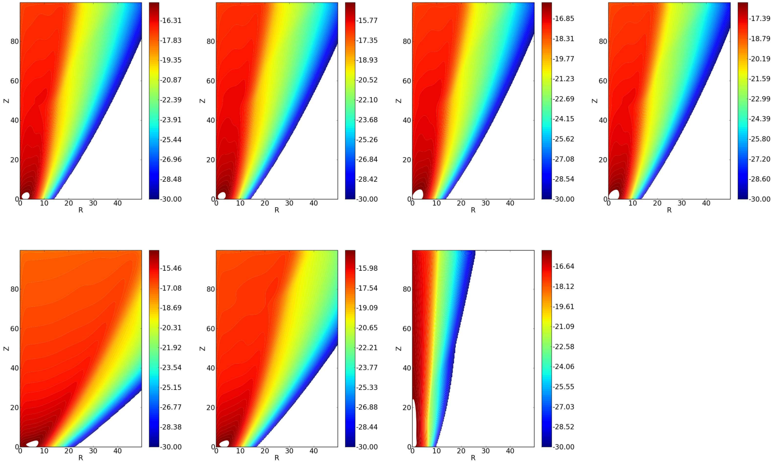
Calculations

- Runs of the pipeline:

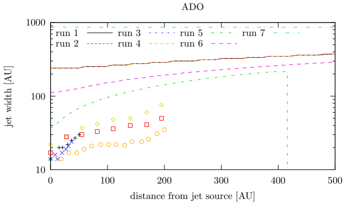
Run	ρ_0 [g cm^{-3}]	v_0 [km s^{-1}]	$\mathcal{M}$ [$M_\odot$]	R_0 [AU]
1	2.6457×10^{-17}	3	2	48.05
2	5.2914×10^{-17}	3	2	48.05
3	1.3229×10^{-17}	3	2	48.05
4	6.6143×10^{-18}	3	2	48.05
5	2.6457×10^{-17}	10	2	4.32
6	2.6457×10^{-17}	5	2	17.30
7	2.6457×10^{-17}	1	2	432.43

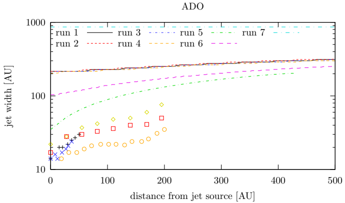
- creation of synthetic emission maps of the [SII] λ6731 and [OI] λ6300 lines
- extraction of the jet width from the maps

Is truncation really needed?



Is truncation really needed?





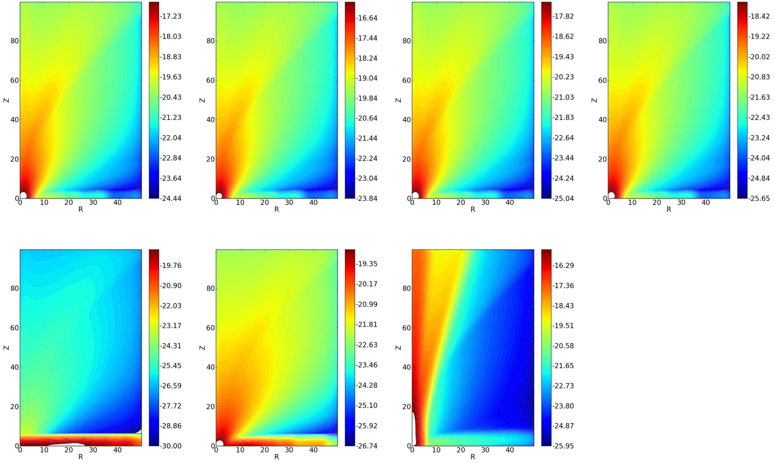
observational data adapted from Ray et al.

(2007)

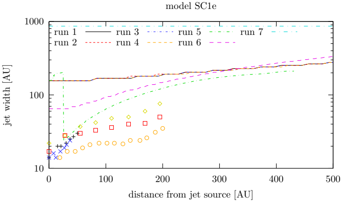
Jet widths for model ADO

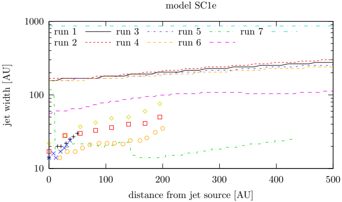
- runs 1–4 are very close together as expected. Since we extract the jet width from a ratio of intensities, the factor ρ^2 cancels out
- in runs 1 and 5–7, where ρ_0 is identical, but v_0 varies between 1 and 10 km s^{-1}, the jet widths decrease monotonically with increasing v_0
- the smallest jet width of all our runs, run 5, is too large by a factor of two with respect to the observations
  $\Longrightarrow$ velocity of about 1400 km s^{-1} or mass of central object of 0.03 or 0.23 $M_\odot$ needed
  $\Longrightarrow$ unrealistic, thus truncation is only option

Effects of truncation



Effects of truncation





observational data adapted from Ray et al.

(2007)

Jet widths for model model SC1e

- same behaviour of runs 1–7 as in for model ADO
- in [OI] λ6300, run 5 is closer than in run 5 for model ADO, lower velocities required wrt to ADO
- in [SII] λ6731, run 5 can reproduce order of magnitude of lowest observed widths (RW Aur)
- in [SII] λ6731, jet velocities between 500 and 1000 km s^{-1} are required in order to explain other objects

$\Longrightarrow$ even more drastical truncation is needed!

Conclusions

Truncated analytical MHD models ...

- ... show shocks at the beginning, transmitting the information of the truncation downstream
- ... are stable afterwards
- ... are needed to explain observed jet widths
- ... are still not truncated enough
  $\Longrightarrow$ to be continued

Thank you for your attention!