Aspect Ratio Dependence in MRI Shearing Box Simulations

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Outline

- Motivation - the MRI
- Shearing Box Model
- Numerical Simulations
- Aspect Ratio dependence
- Conclusions

Motivations

- Turbulence possible source of angular momentum transport in accretion disks (Shakura & Sunyaev, 1973)
- Microscopic viscosity too small.
- Magnetorotational instability (MRI, Velikhov 1959, Chandrasekhar 1960) re-discovered by Balbus & Hawley (1991) proposed as the main process at the base of angular momentum transport in accretion disks.

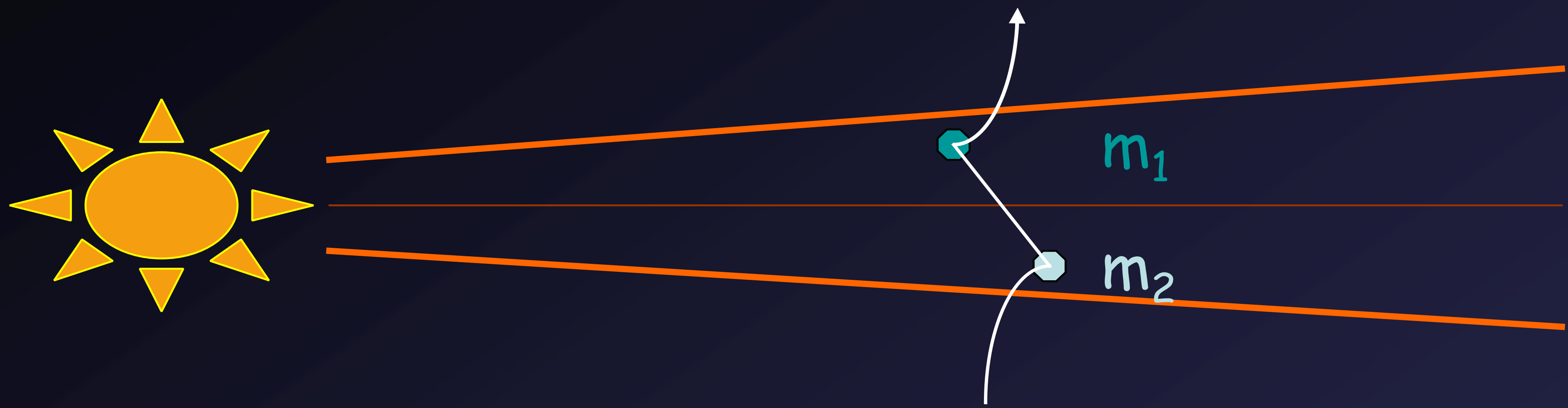
The MRI

- MRI is a fluid instability whereby fluid elements exchange angular momentum via distorted field lines.
- It is an intrinsic MHD instability.
- Consider two fluid elements joined by the same field line:
- Magnetic field acts as a "spring" connecting two neighboring fluid elements:

- If perturbed, the line stretches and develops tension…

The MRI

- If $\partial\Omega/\partial r < 0$



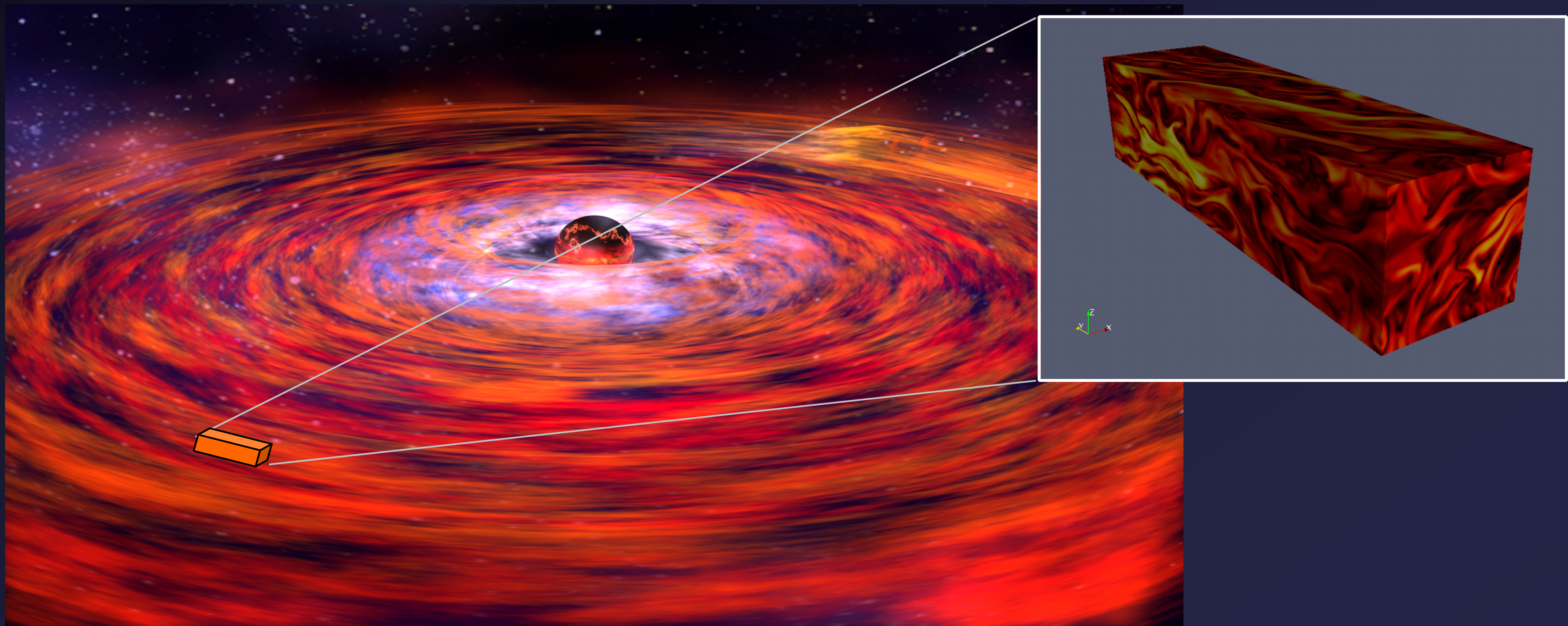
- The tension acts to reduce the angular momentum of m_1 and increase that of m_2:
  - inner elements are forced to "slow" down, reduce angular momentum and move to a lower orbit;
  - outer fluid elements "speed up", increase angular momentum and move to higher orbit;
- This further increases the tension and the process "runs away".

Important Facts…

- Full, global disk models extremely challenging and beyond present capabilities.
- → Local approaches allow to reach much higher resolutions;
- Meaningful if they capture the characteristic of the MRI driven turbulence and angular momentum (AM) transport.
- Validity should be verified *a posteriori,* by checking that solution does not depend on size of computational domain.
- Much of what is presently known about MRI comes from shearing box simulations.
- SB may be questionable (Jim Stone Talk, Regev & Umurhan 2008 A&A)

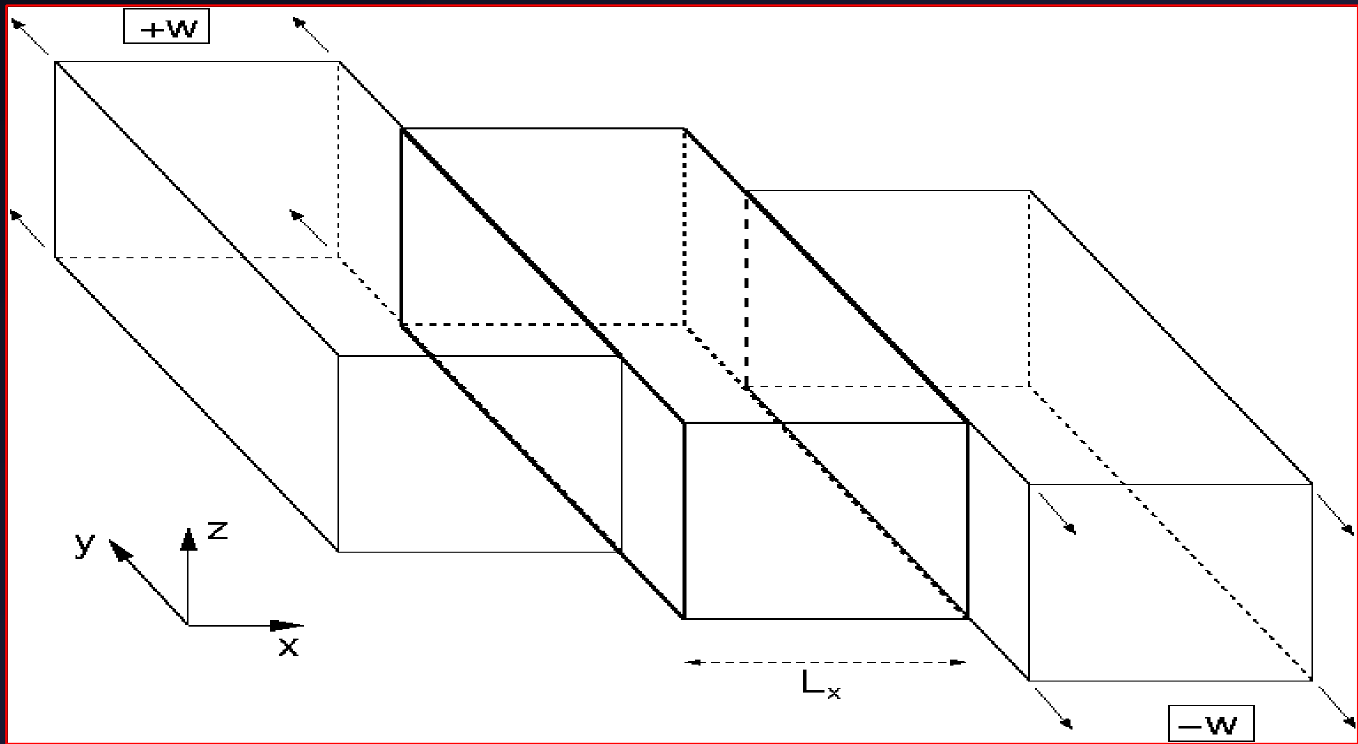
The Shearing Box Approximation

- Local models of differentially rotating systems appeal to the shearing box approximation, based on a local expansion of the tidal forces in a reference frame corotating with the disk at some fiducial radius R_0.



The Shearing Box Approximation

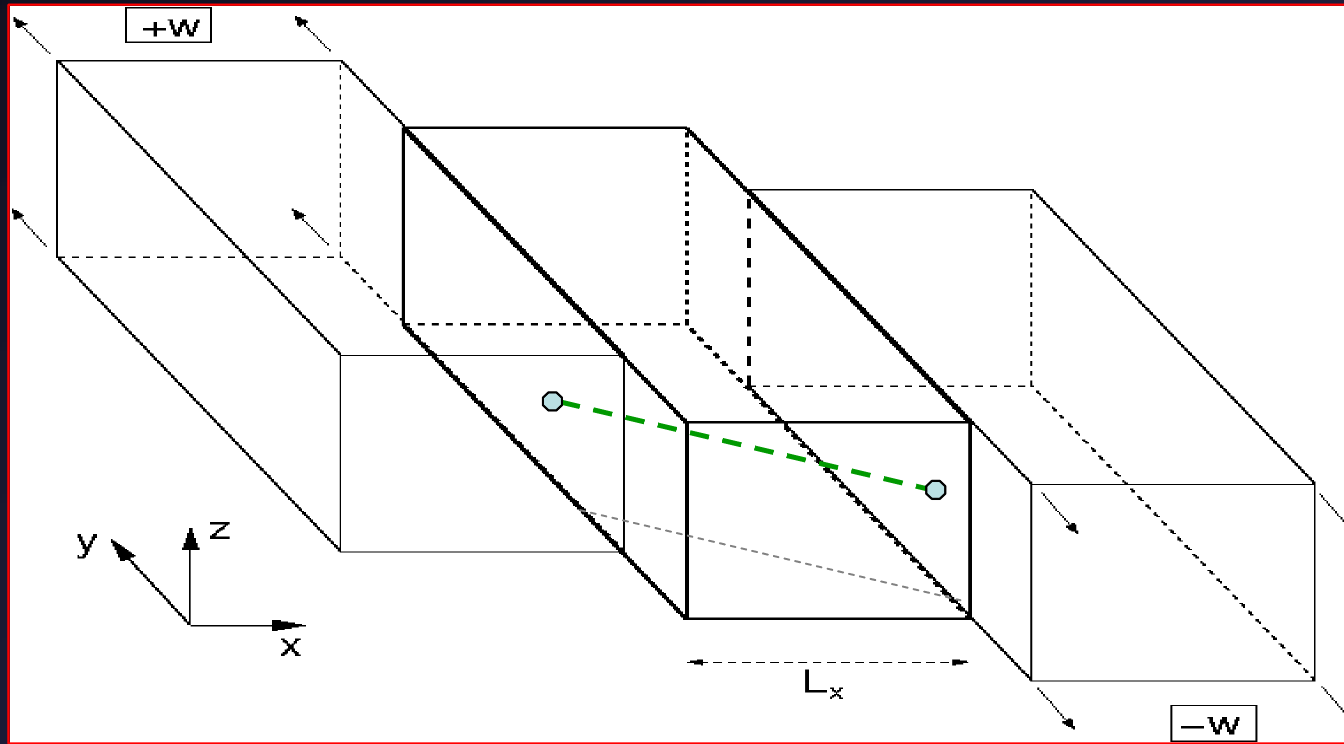
- The validity of the approximation is restricted to a small Cartesian box with a steady flow consisting of a linear shear velocity, normally considered as the basic flow.



- While the computational box is periodic in the azimuthal (y) and vertical (z) directions, radial (x) boundary conditions are determined by “image” boxes sliding relative to the computational domain.

Boundary Conditions

- ***Vertical (z) direction***:
  periodic / open
- ***Azimuthal (y) direction***:
  periodic
- ***Radial (x) direction***:
  Shearing boundary



$$
\begin{aligned}
q(x,y,t) &= q\left(x+L_x, y-wt, t\right) \\
v_y(x,y,t) &= v_y\left(x+L_x, y-wt, t\right)+w
\end{aligned}
$$

left

$$
\begin{aligned}
q(x,y,t) &= q\left(x-L_x, y+wt, t\right) \\
v_y(x,y,t) &= v_y\left(x-L_x, y+wt, t\right)-w
\end{aligned}
$$

right

Numerical Issues

- By exploiting the periodicity in the y- and z- directions, volume-integrated quantities should be "conserved" under advection:

$$\boxed{\int \left(\frac{\partial Q}{\partial t} + \nabla \cdot \vec{F}\right) d^3x = 0 \quad \Longrightarrow \quad \Delta\mathcal{V}\frac{d\left\langle Q\right\rangle}{dt} = \int \left[F_x(x_L, y, z) - F_x(x_R, y, z)\right] dy\, dz}$$

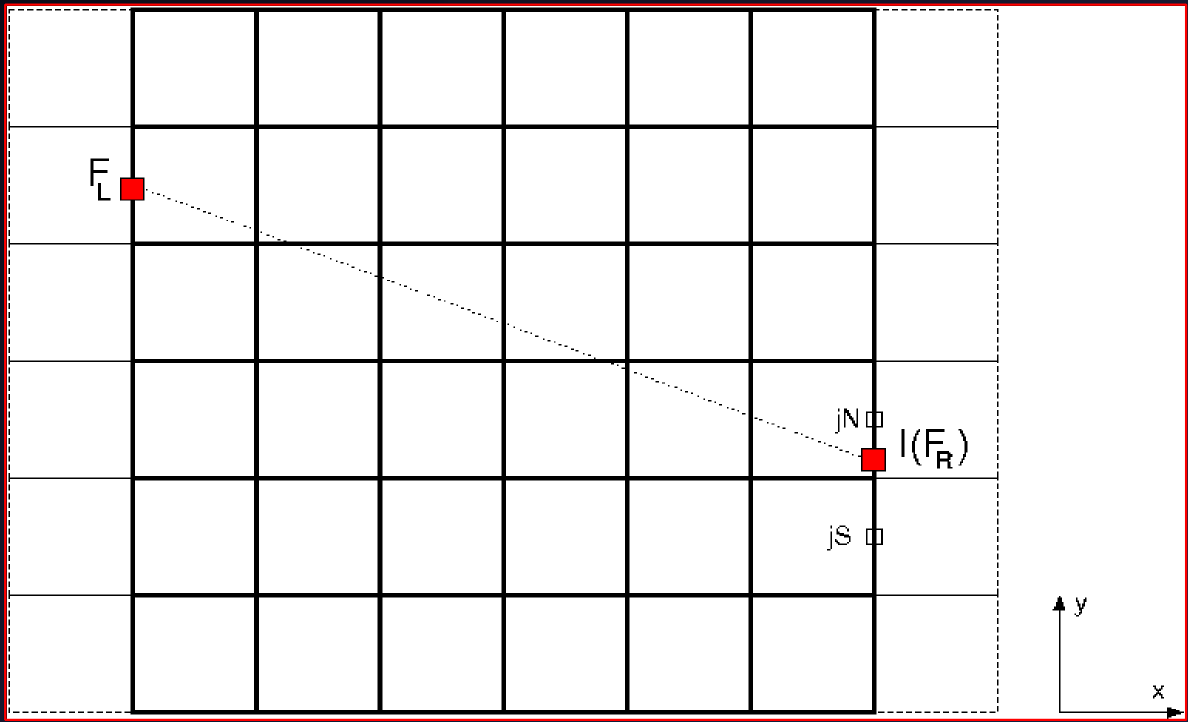
- Strict conservation applies only to mass, z-momentum, x and z component of the magnetic field. For the y-component of B, the following relation must be satisfied:

$$\boxed{\Delta\mathcal{V}\frac{d\left\langle B_y\right\rangle}{dt} = w\int B_x(x_L, y, z)\, dy\, dz \equiv w\int B_x(x_R, y, z)\, dy\, dz}$$

- *However, since the numerical fluxes are nonlinear functions of the Q's, truncation errors introduced by the interpolation algorithm in the radial ghost zones can lead to significant deviations from conservation.*

Remapping Procedure

- Loss of conservation can be avoided by properly matching computed x-fluxes at the sheared domain boundaries:



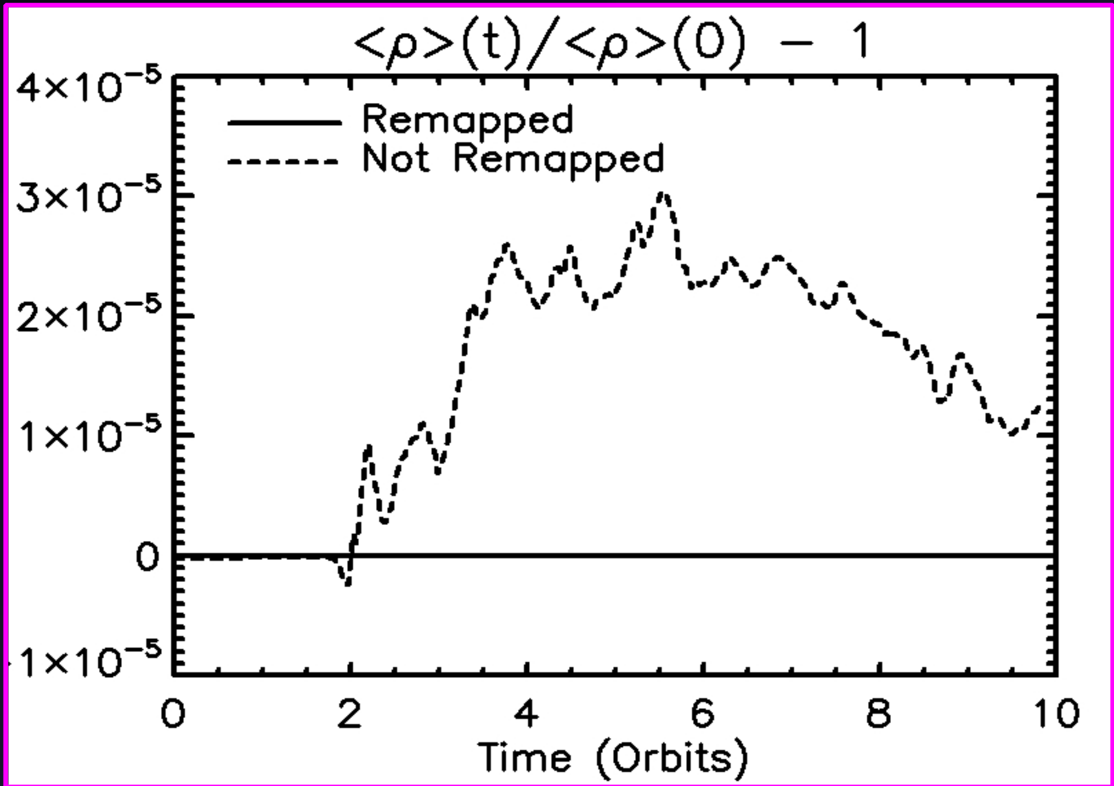
Fluxes not containing the azimuthal velocity (vy) are easily symmetrized by

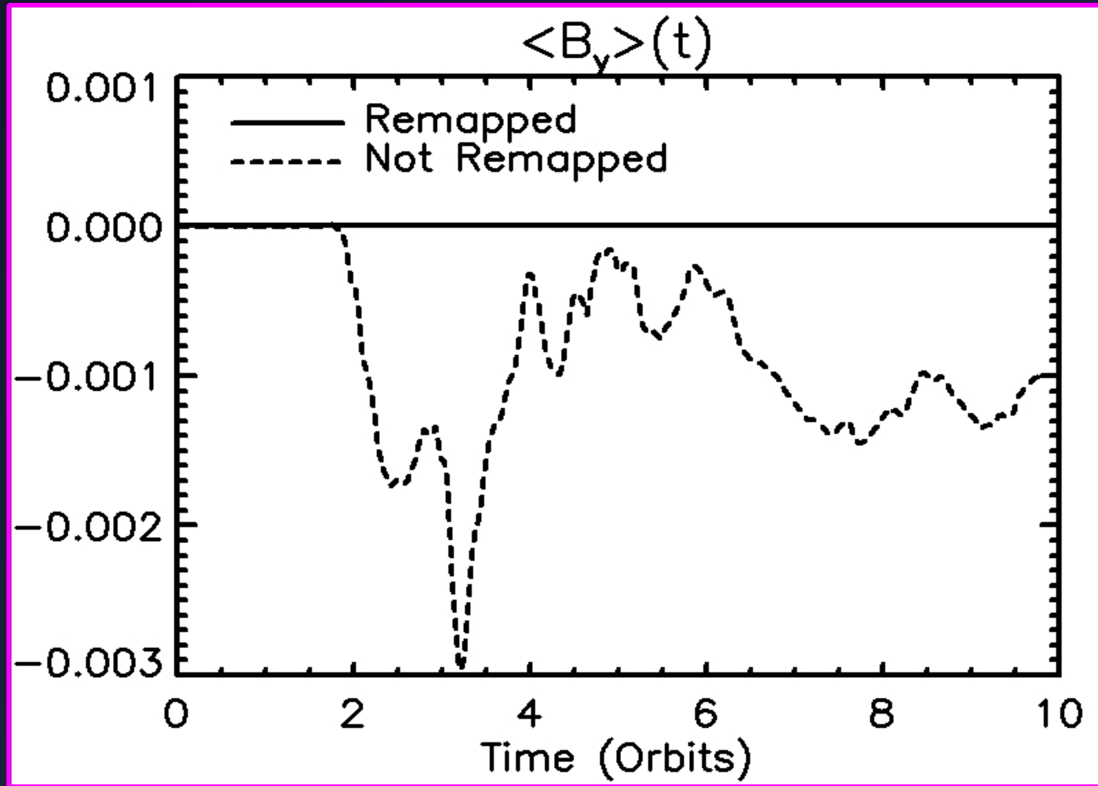
$$F_L \to \frac{1}{2}\left[F_L + \mathcal{I}\left(F_R\right)\right] \;, \quad F_R \to \frac{1}{2}\left[F_R + \mathcal{I}\left(F_L\right)\right]$$

Ziegler (2007), Mignone (2008), in prep.

- Additional care is taken to ensure magnetic flux conservation by suitable remapping of the E.M.F z-component.

Remapping Procedure





Shearing Sheet Equations

- If the box is sufficently small, curvature terms can be neglected and a local system of Cartesian coords is adopted

  (r – r0) → x, ϕ → y, z → z:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} + 2\mathbf{\Omega}\times\mathbf{v} = \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi\rho} - \frac{1}{\rho}\nabla\left(\frac{\mathbf{B}^2}{8\pi} + \mathbf{p}\right) - \nabla\left(2\mathbf{A}\mathbf{\Omega}\mathbf{x}^2 + \cancel{\frac{1}{2}\mathbf{\Omega}^2\mathbf{z}^2}\right)$$

- Where $A = \frac{R}{2}\left(\frac{\partial\Omega(R)}{\partial R}\right)_{R_0}$ is the shear rate.

- Keplerian profile: $A = -\frac{3}{4}\Omega$

- Velocity profile: $v_{y0} = -2Ax$

Exact Nonlinear MRI mode

- Goodman & Xu (1994) showed the existence of exact exponentially growing solutions of shearing sheet incompressible equations (channel solutions):

$$(B_x, B_y, B_z) = \epsilon B_0 \exp(st) \cos(Kz)(\sin\gamma, -\cos\gamma, 0) + (0, 0, B_0)$$

$$(v_x, v_y, v_z) = \epsilon v_0 \exp(st) \sin(Kz)(\cos\gamma, \sin\gamma, 0) + (0, 2Ax, 0)$$

- K: wave number (K || rot. Axis)
- γ: angle between the magnetic field and the y direction
- B_0 background axial field, $v_0 = -2A/K \sin^2\gamma$
- Velocity and mag. field fluctuations orthogonal
- s (growth rate) = -A sin(2γ)

Exact Nonlinear MRI mode

- $s>0 \rightarrow$ exponentially growing solutions
- $\gamma = \pi/2 \rightarrow$ field radial, $s = 0$
- Fastest growing mode has $\gamma = \pi/4$, $K \approx \Omega/V_A$

STILL:

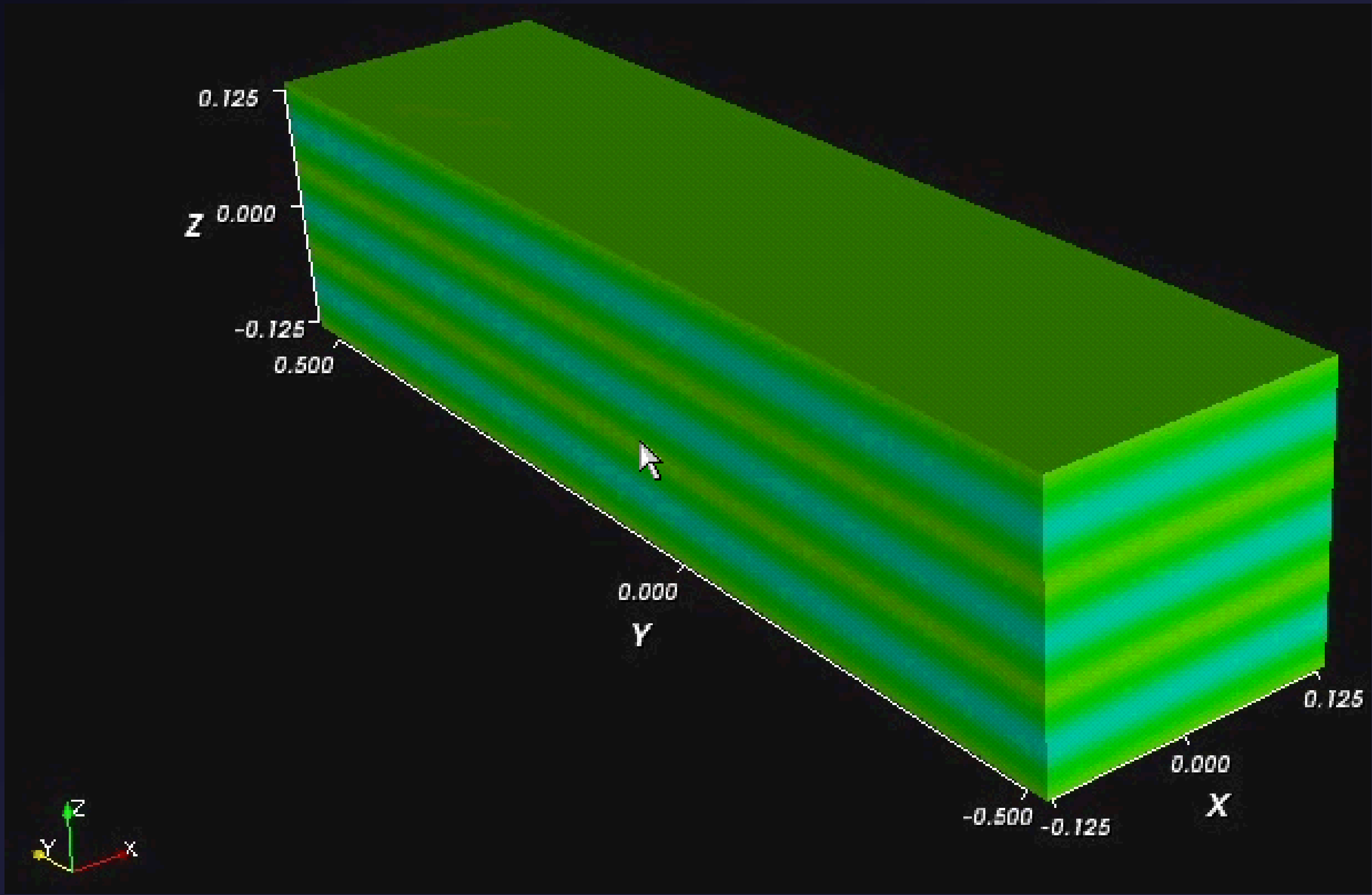
- The nonlinear exact mode is itself linearly unstable to secondary *"parasitic" instabilities*;
- The growth rate $\propto$ MRI amplitude;
- They may stop MRI growth

Numerical Simulations

- Wish to study how the properties of turbulent solution vary as the box aspect ratio changes;
- Compressible, isothermal 3D simulations of shearing box with no explicit dissipation;
- Neglect vertical stratification and gravity
- PLUTO code (Mignone et al. 2007):
  - Conservative code
  - 2nd order accuracy in space and time;
  - Corner Transport Upwind (Colella ’90);
  - Constrained transport for magnetic field evolution

Numerical Simulations

- Parameters:

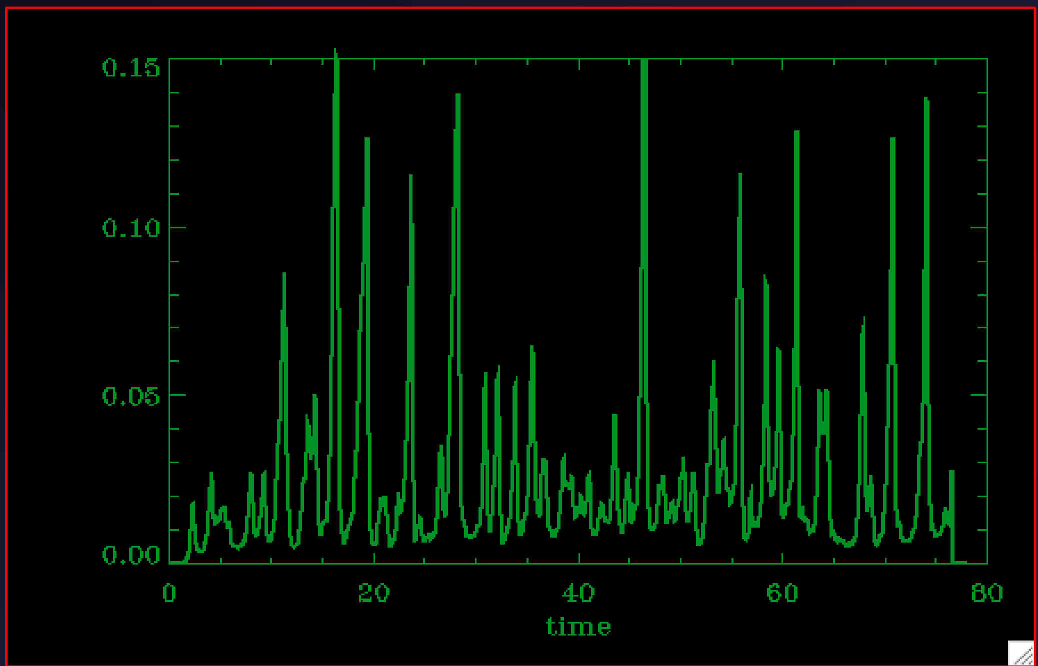


- $\beta = 10^4$
- cs = 4.56
- $2\pi/K = 1/3$
- Lx:Ly:Lz = L:4:1
  L (aspect ratio) =1,4,8

- Resolution: 32-64-128

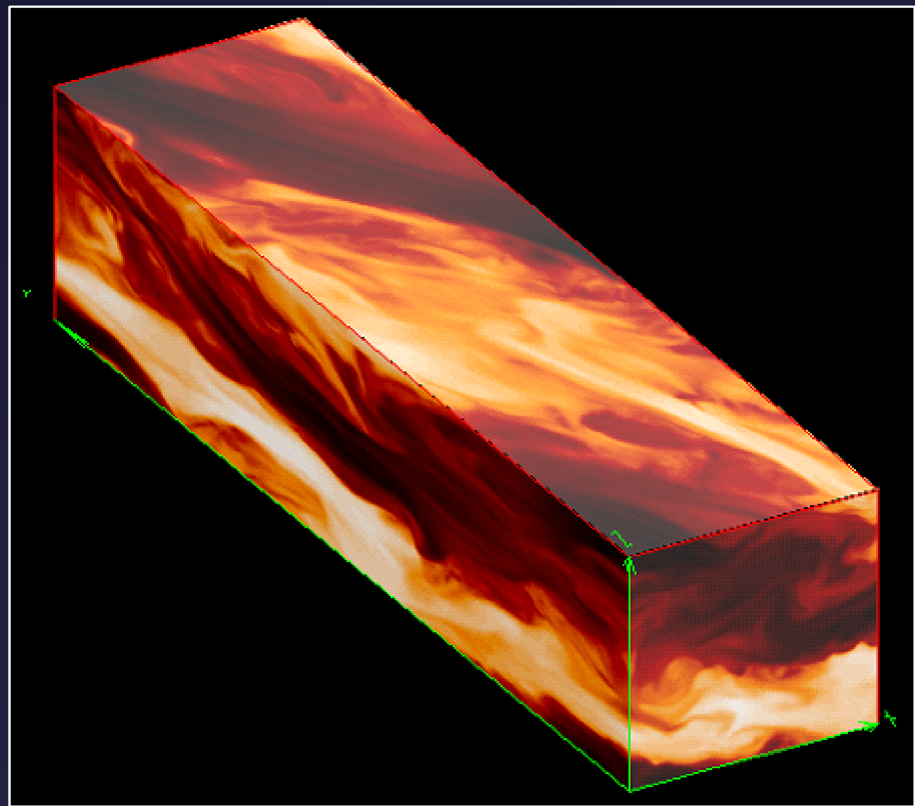
"Channel Solution"

- L = 1 → intermittent behavior with episodes of high AM transport.
- "Channel Solution" → behavior similar to exact solution
- 1st peak in the curve → channel solution with wavelength 1/3 of vertical box size (mode with maximum growth rate). Once disrupted by secondary instabilities, only the 1-channel appears.



Volume average maxwell stress

$$\langle w_{xy} \rangle = -\langle B_x B_y \rangle$$

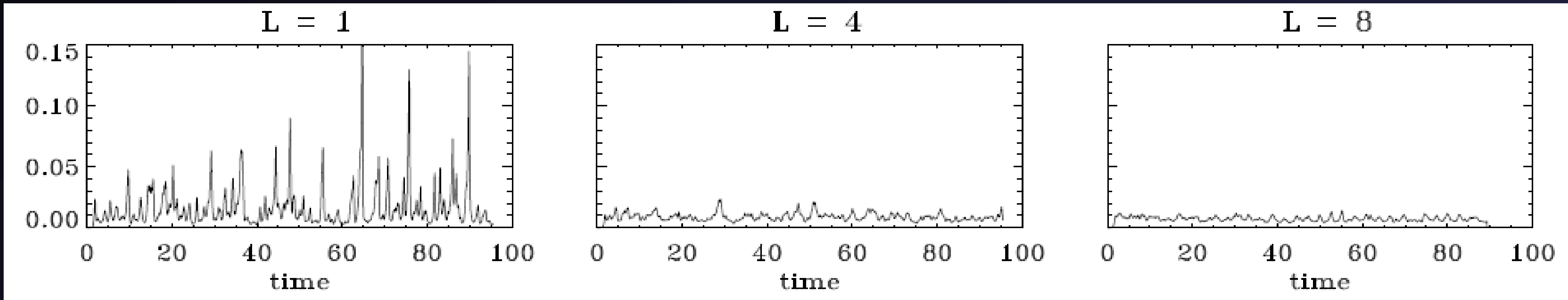


Properties of the Channel Solution

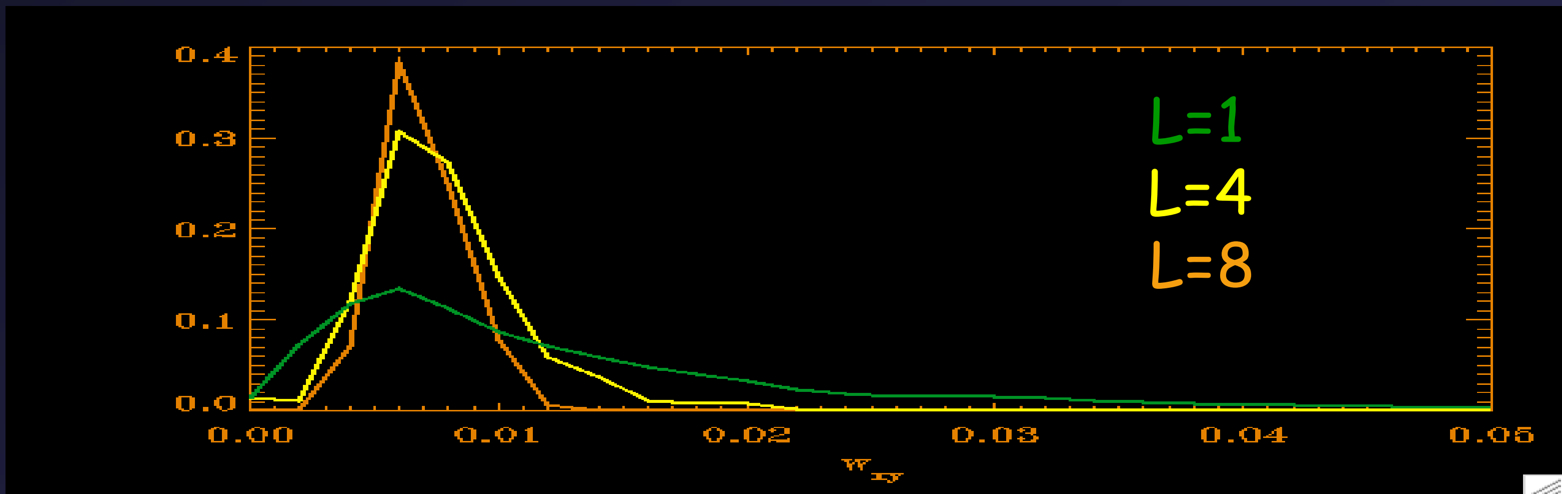
- Channel solution: highly correlated state for which the AM transport is very efficient.
- Peaks of Maxwell stress → high AM transport.
- High correlation coefficients between directional components of magnetic field and velocity perturbations.
- Double-peaked distribution functions.
- Disruption by parasitic instabilities.
- Channel solution with wavelength equal to vertical box size appear to be a dominant feature of the shearing box MRI simulation with nonzero net flux.

Maxwell Stresses

- What happens when we change L ?



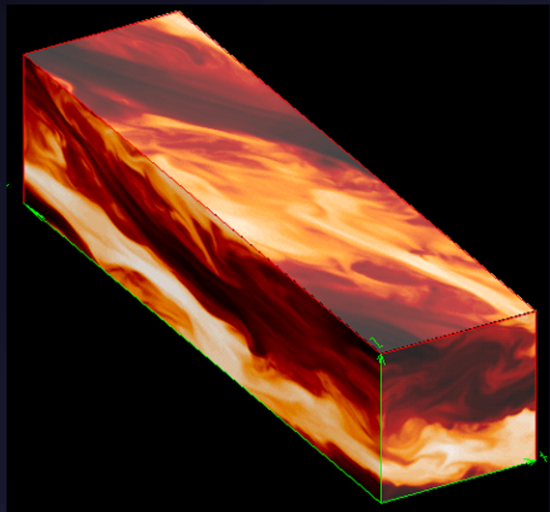
- Spikes absent
- Confirmed by probability distribution function → long tail correspond to episode of enhanced transport

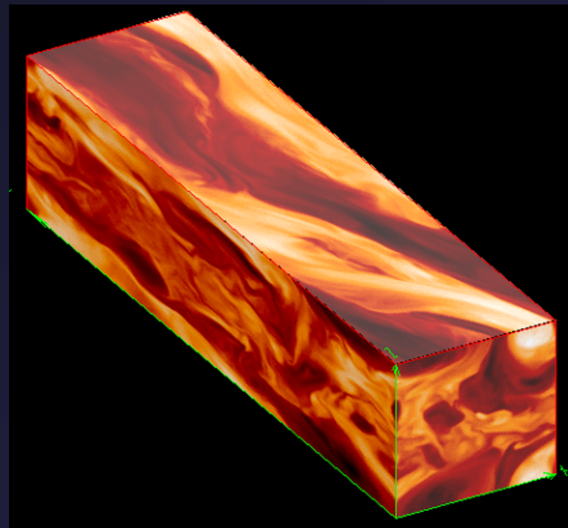


Distribution between peaks

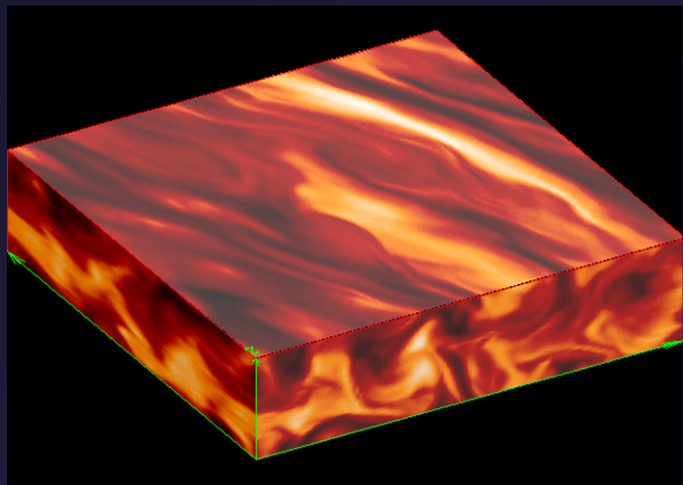
Max | Min

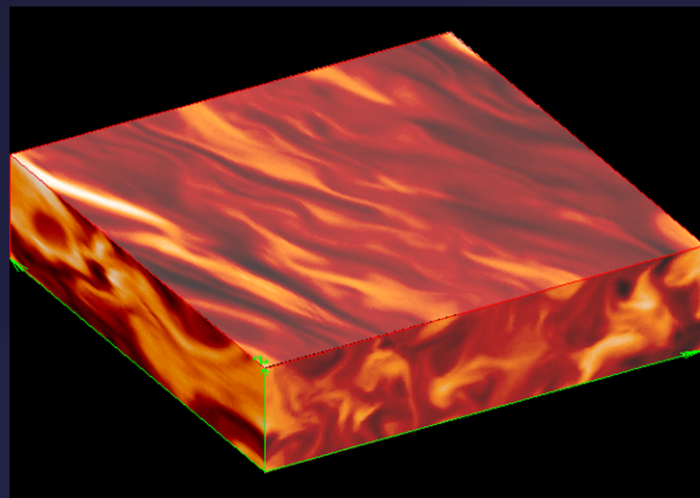
L=1





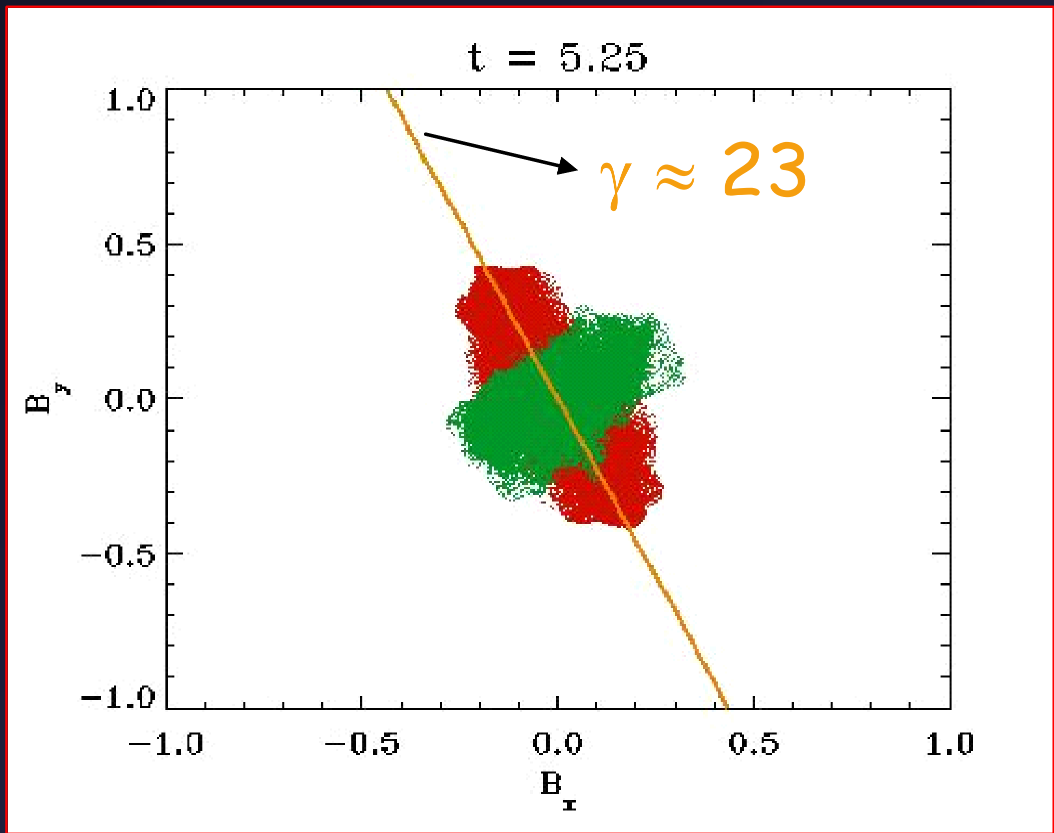
L=4





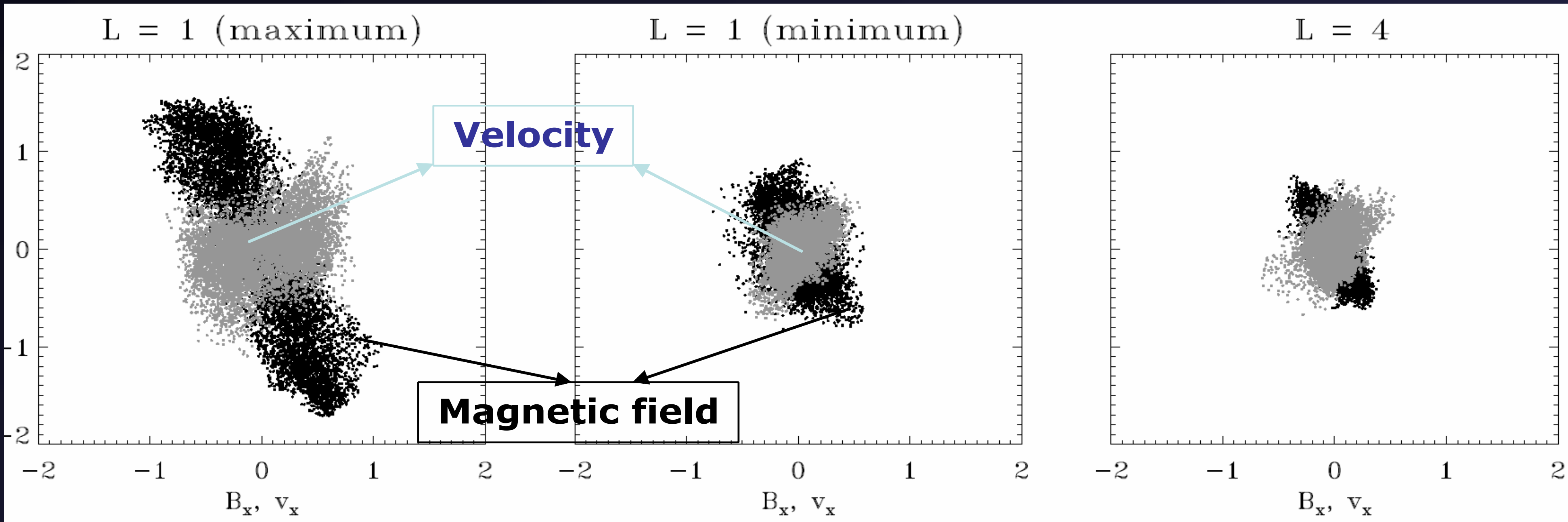
Bx - By Correlation

- For a pure channel solution, Bx and By lie on a straight line with slope tan(γ);
- In general → scatter plot of of B_y vs B_x for a maximum



Bx - By Correlation

- Scatter plot for different aspect ratios:

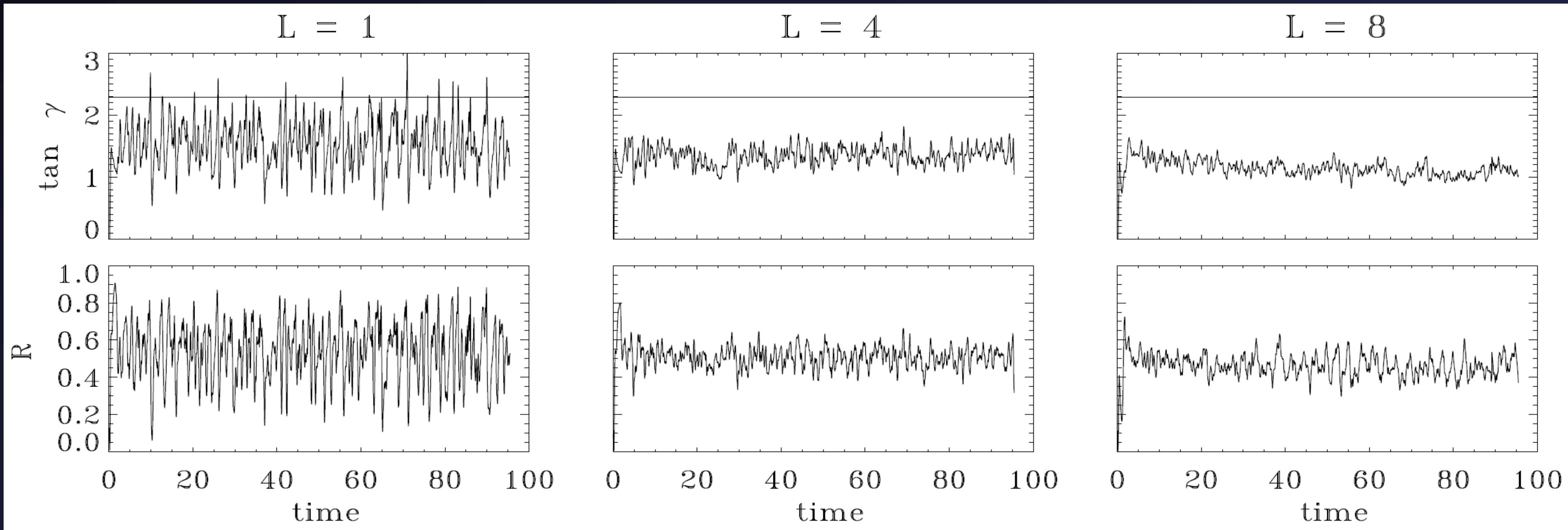


- L=4: no significant variations → similar to minimum state of case L =1.
- Reynold stresses are lower beacuse vel. fluctuations are smaller and remain uncorrelated.

Bx - By Correlation

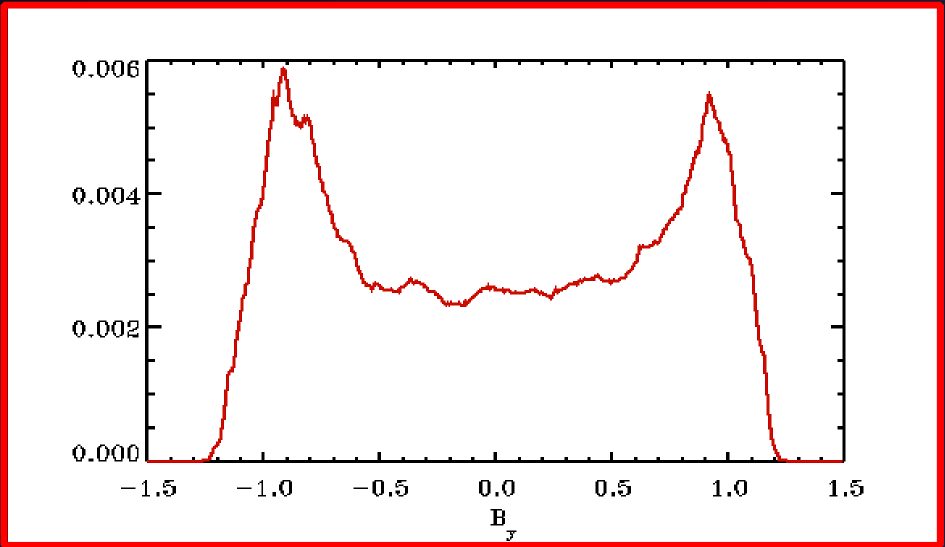
- Least-square Line fit to define average angle and correlation coefficients R:

$$\tan\gamma = -\frac{\langle B_x B_y\rangle}{\langle B_x^2\rangle}, \quad \text{and} \quad R = \frac{\langle B_x B_y\rangle^2}{\langle B_x^2\rangle\langle B_y^2\rangle}.$$

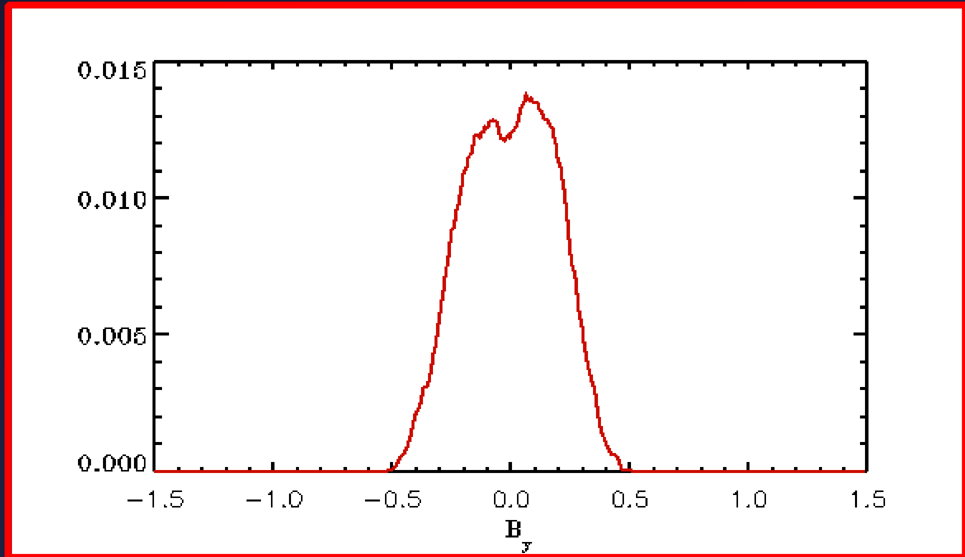


Probability Distribution Function

- Probability distribution function of the azimuthal component (B_y) have almost gaussian distributions in correspondence of the minima and very distorted distributions with "double peaks" in correspondence of the maxima.
- The presence of the double peak → presence of channel solution



maximum

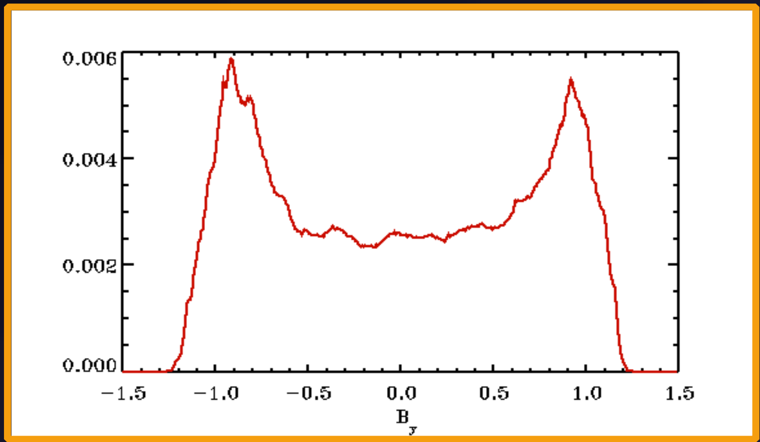


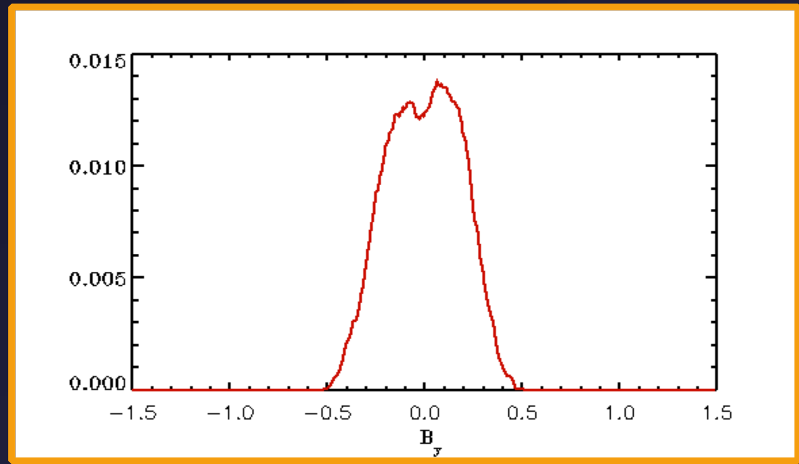
minimum

Probability Distribution Function

maximum *minimum*

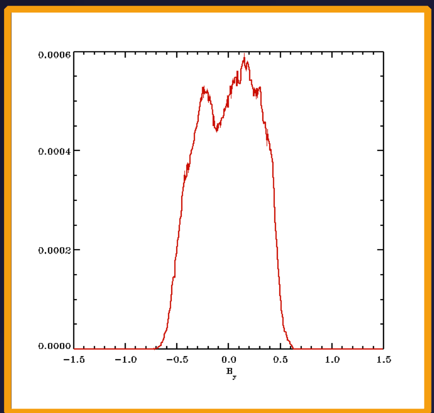
L = 1

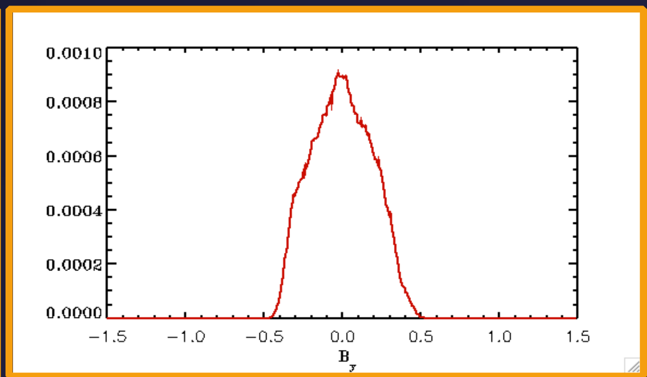


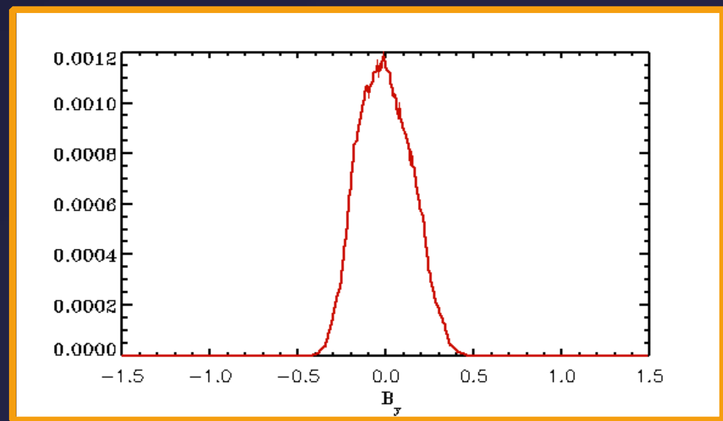


Largest **Typical**

L = 4





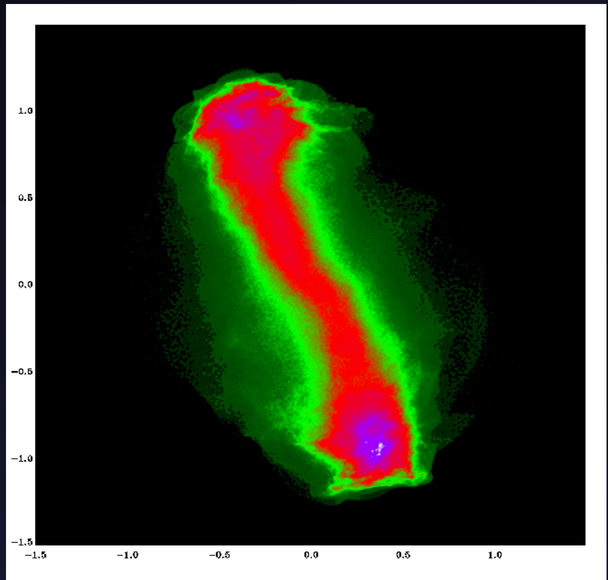


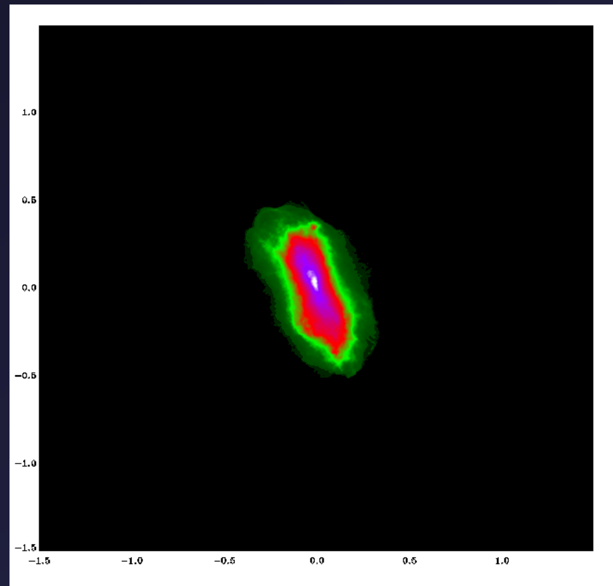
2D Distribution (Bx – By)

maximum

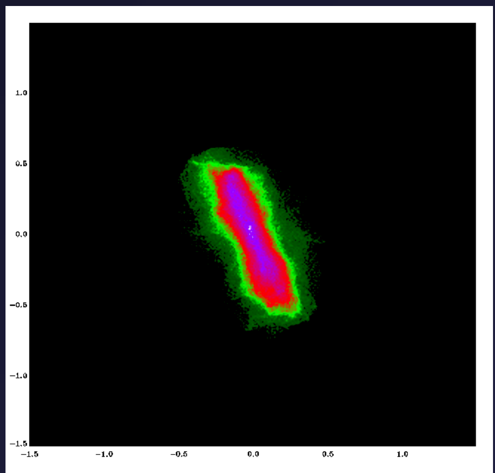
minimum

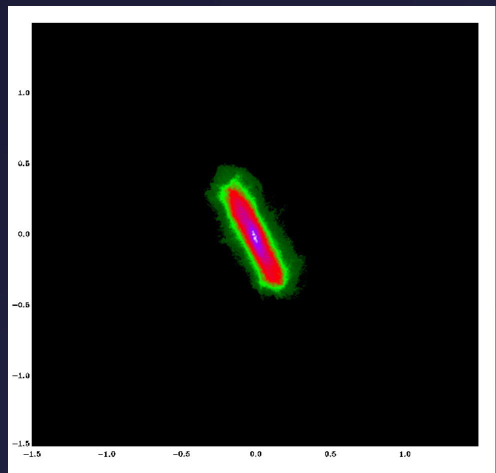
L = 1

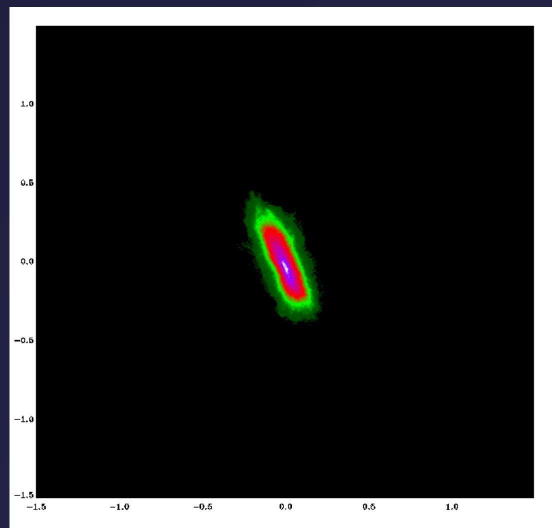




L = 4







Conclusions

- 3D, compressible simulation of Shearing Box MRI;
- Box size: L:4:1, L = “aspect ratio”;
- Behavior of the system depends on the aspect ratio L of the box;
- L=1, peaks correspond to the formation of a channel solution;
- Increasing the aspect ratio, the system has more difficulty in forming the channel:
  - System remains in state with lower correlation;
  - Transport is inhibited
- Going from L=4 to L=8 seems to indicate a tendence towards convergence;

Conclusions

- Possible explanation: parasitic instabilities have wavelength larger than vertical size of channel solution (Goodman & Xu 1994). In box with L=1 they may simply be stable;

- Dominance of the channel seems peculiarity induced by an overly constrained geometry;

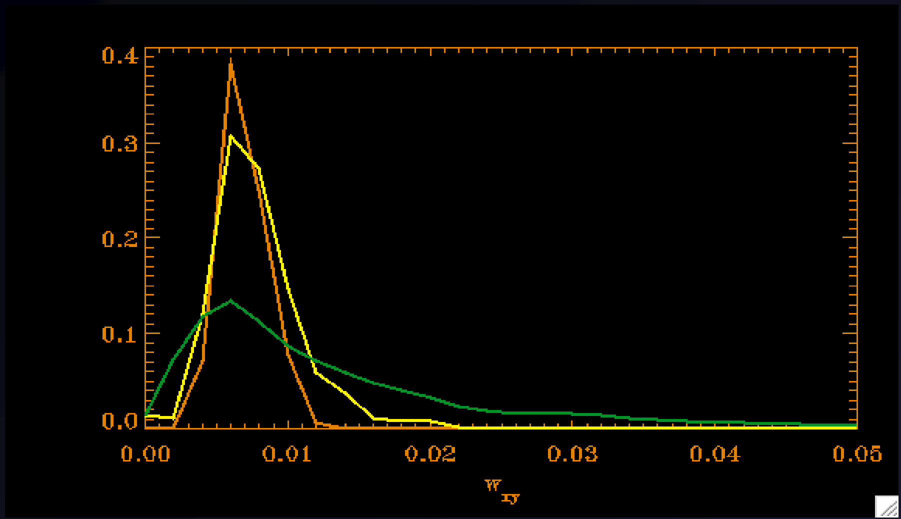
- Shearing box results may be siginificant for full disk only with large aspect ratio;

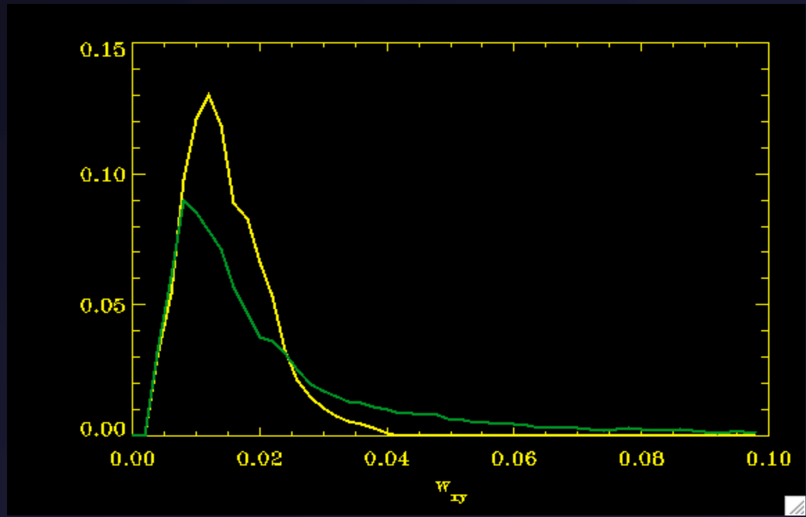
- Increase in resolution introduces significant changes but does not alter the result qualitatively;

- Applicability of SB to full disk has to be tested against global simulations, that may soon be feasible.

Thank You

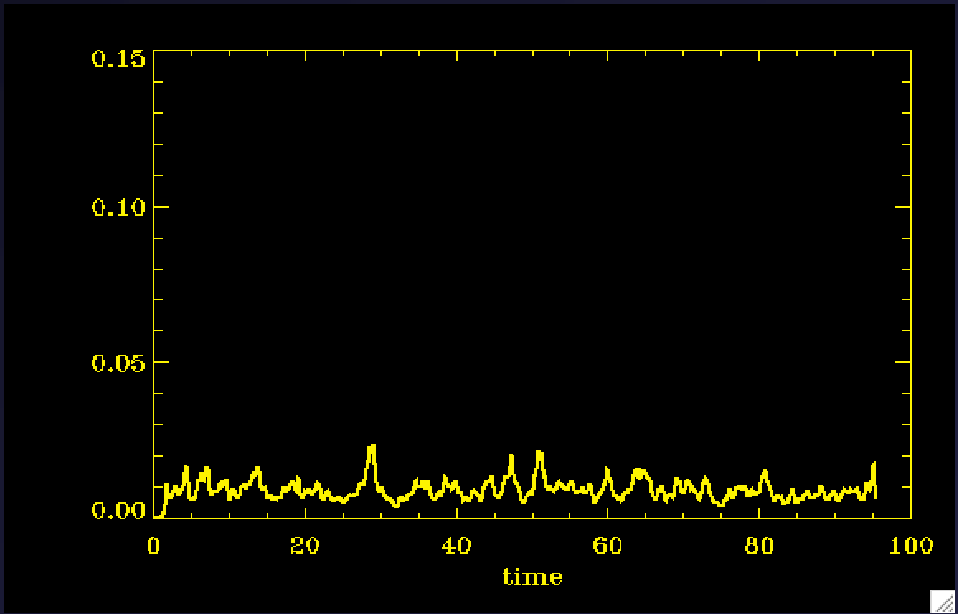
Distribution function of maxwell stresses





Maxwell stresses vs time

Asp. Ratio = 4



Asp. Ratio = 8