Assembling the Ingredients for a Jet:

How do large-scale magnetic fields get there and what happens when it does?

Rothstein & Lovelace (2008, ApJ, in press)

Bisnovatyi-Kogan & Lovelace (2007, ApJ)

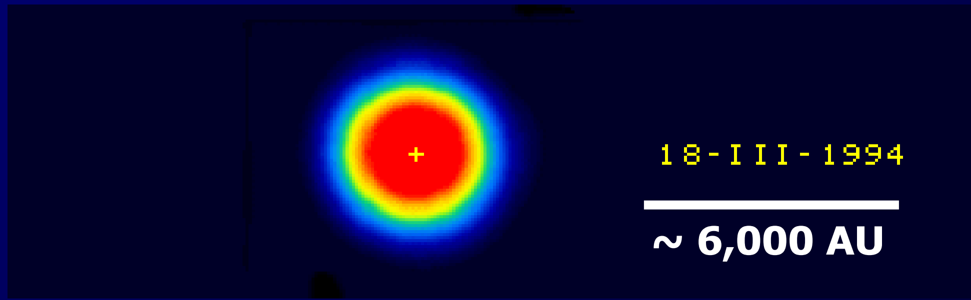
Richard Lovelace

Cornell University

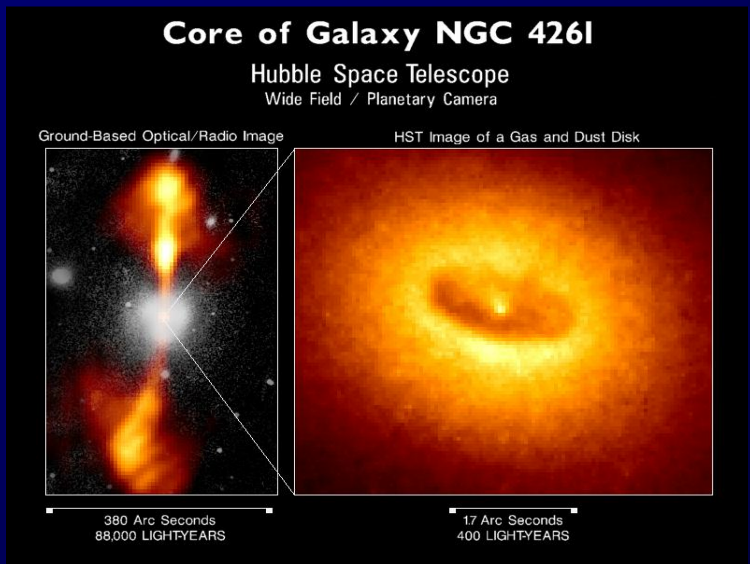
Rhodes, 7 July 2008

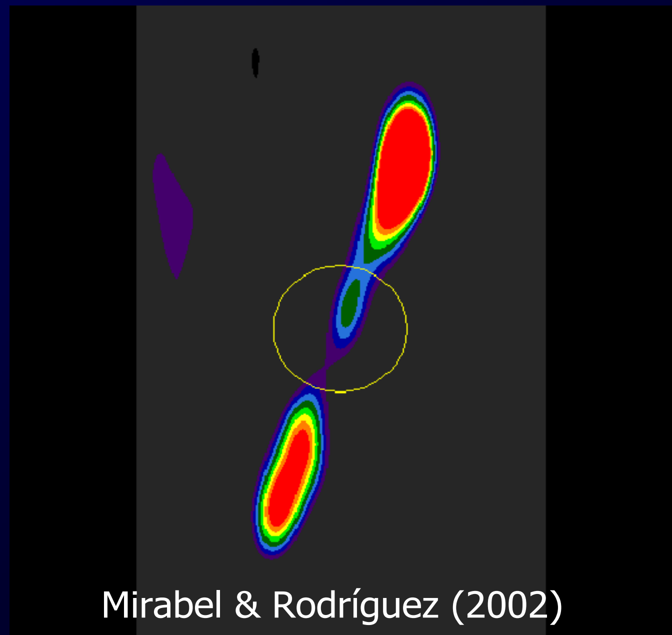
Observations of Jets
(a very concise summary)

There are a lot of them.



Mirabel & Rodríguez (1994)





Mirabel & Rodríguez (2002)

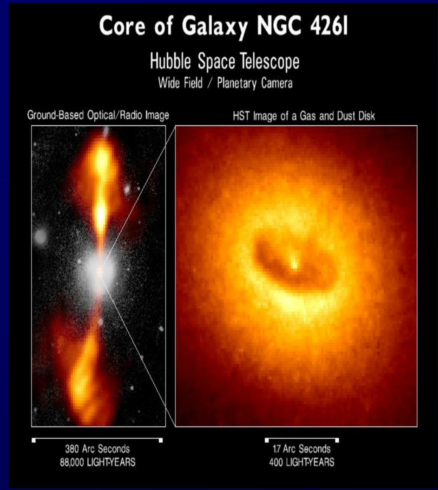
Theoretical Models of Jets
(a very concise summary)

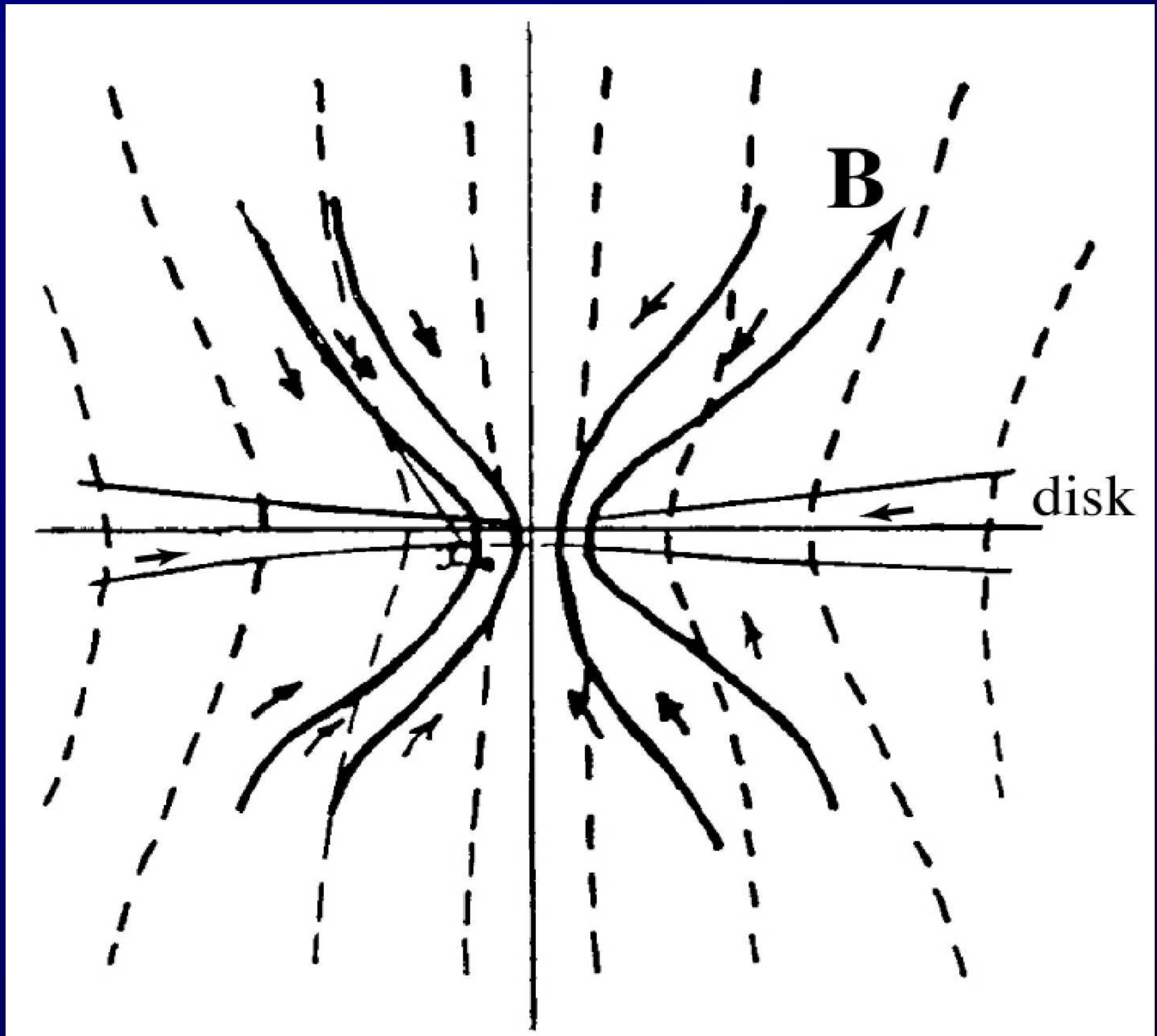
Require large-scale magnetic fields.

(e.g., Lovelace 1976; Blandford 1976; Blandford & Znajek 1977; Blandford & Payne 1982)

The picture you might have in your head

(dates from the 1970's)





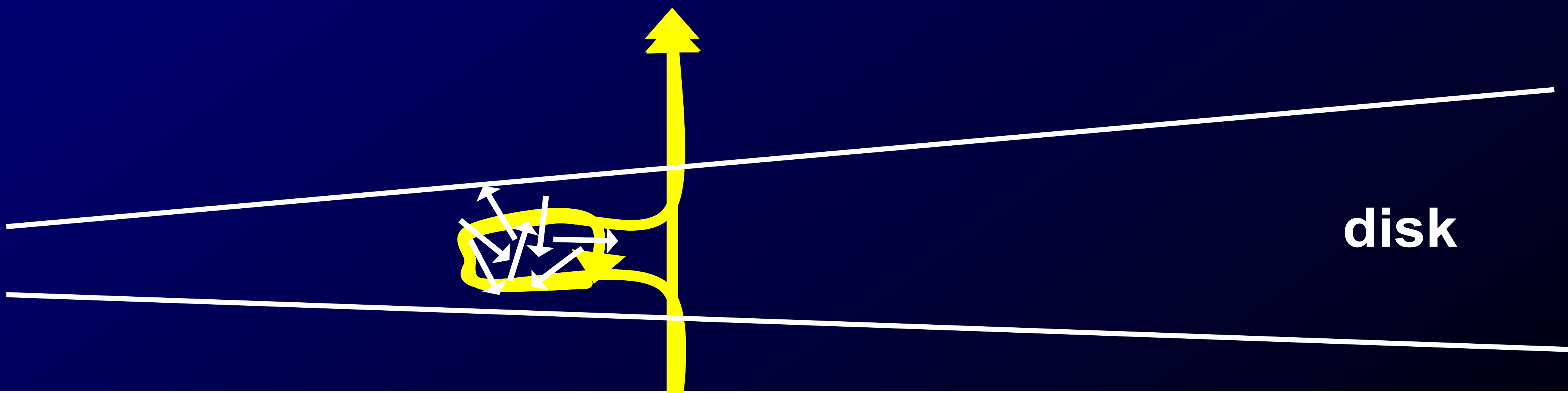
Advection and compression of a weak field leads to a strong field in the inner disk.

Lovelace 1976

But problems with this idea were discovered in the 1990's

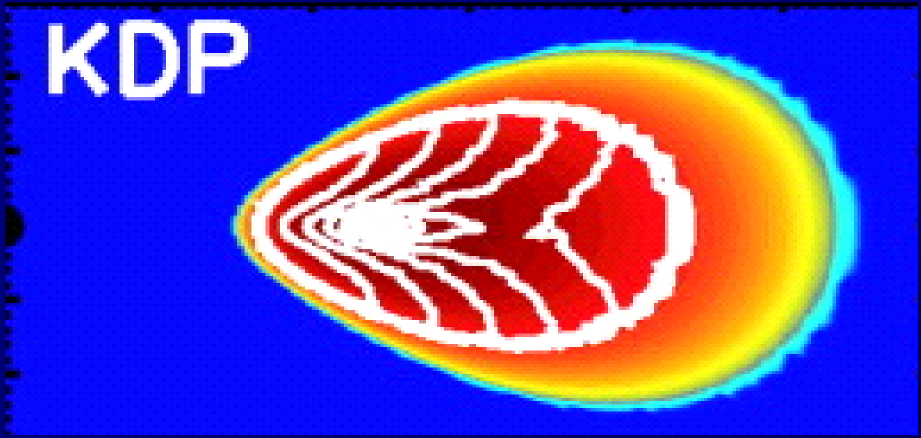
- Turbulence responsible for accretion also leads to enhanced reconnection (field diffusivity)
- A turbulent disk is a poor conductor
- Weak fields cannot advect inward in a thin disk

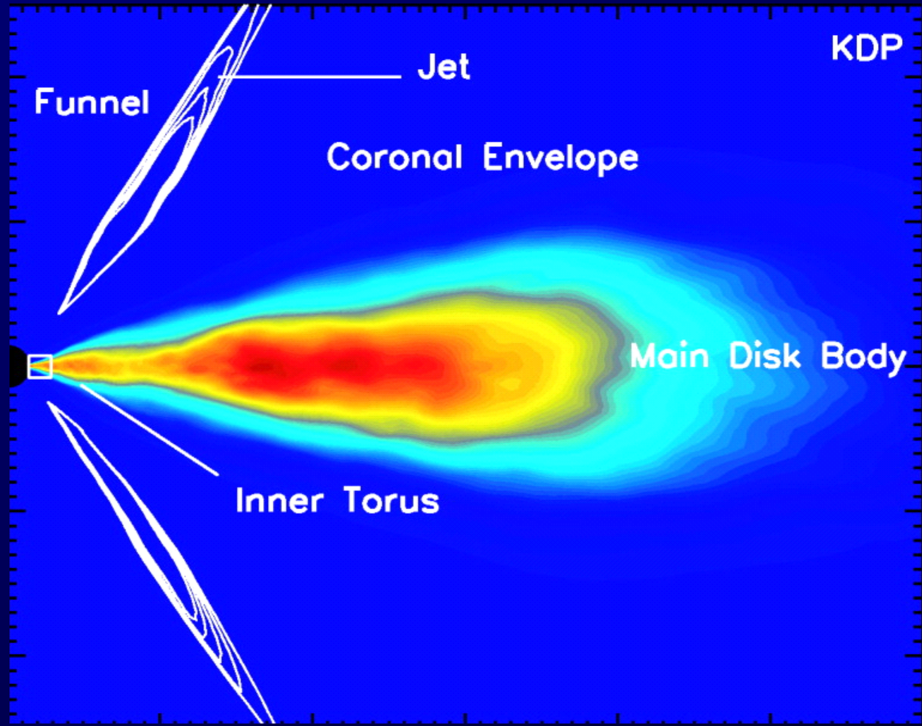
e.g., van Ballegooijen 1989; Lubow et al. 1994; Lovelace et al. 1994; Heyvaerts et al. 1996



Do Numerical Simulations Address This?

(e.g. De Villiers et al. 2003)





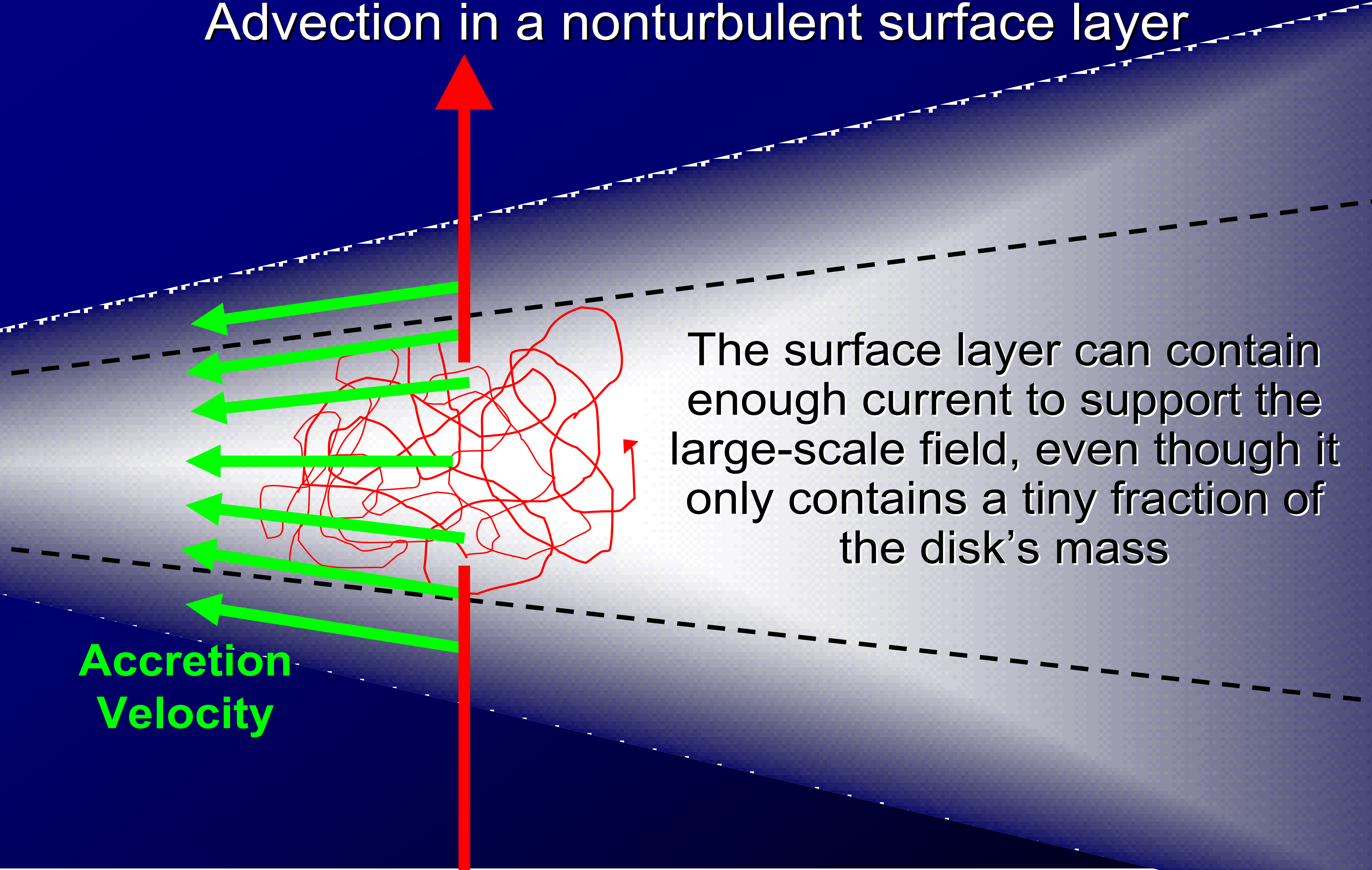
- Limited geometries studied → jet confined to extreme inner disk (black hole-specific effects)
- Interestingly, choice of initial field geometry still seems to have a significant effect on jet properties (McKinney & Gammie 2004; Beckwith et al. 2007)

How might weak magnetic fields be able to advect inward?

- Could be achieved with a mathematically favorable vertical profile of the diffusivity (Ogilvie & Livio 2001)
- Physical model: *Assuming* the surface layer of the disk is nonturbulent (i.e., highly conducting), it can support inward field advection (Bisnovatyi-Kogan & Lovelace 2007)
- Our work: This is likely to occur in physically realistic disks (with magnetorotational turbulence; MRI)

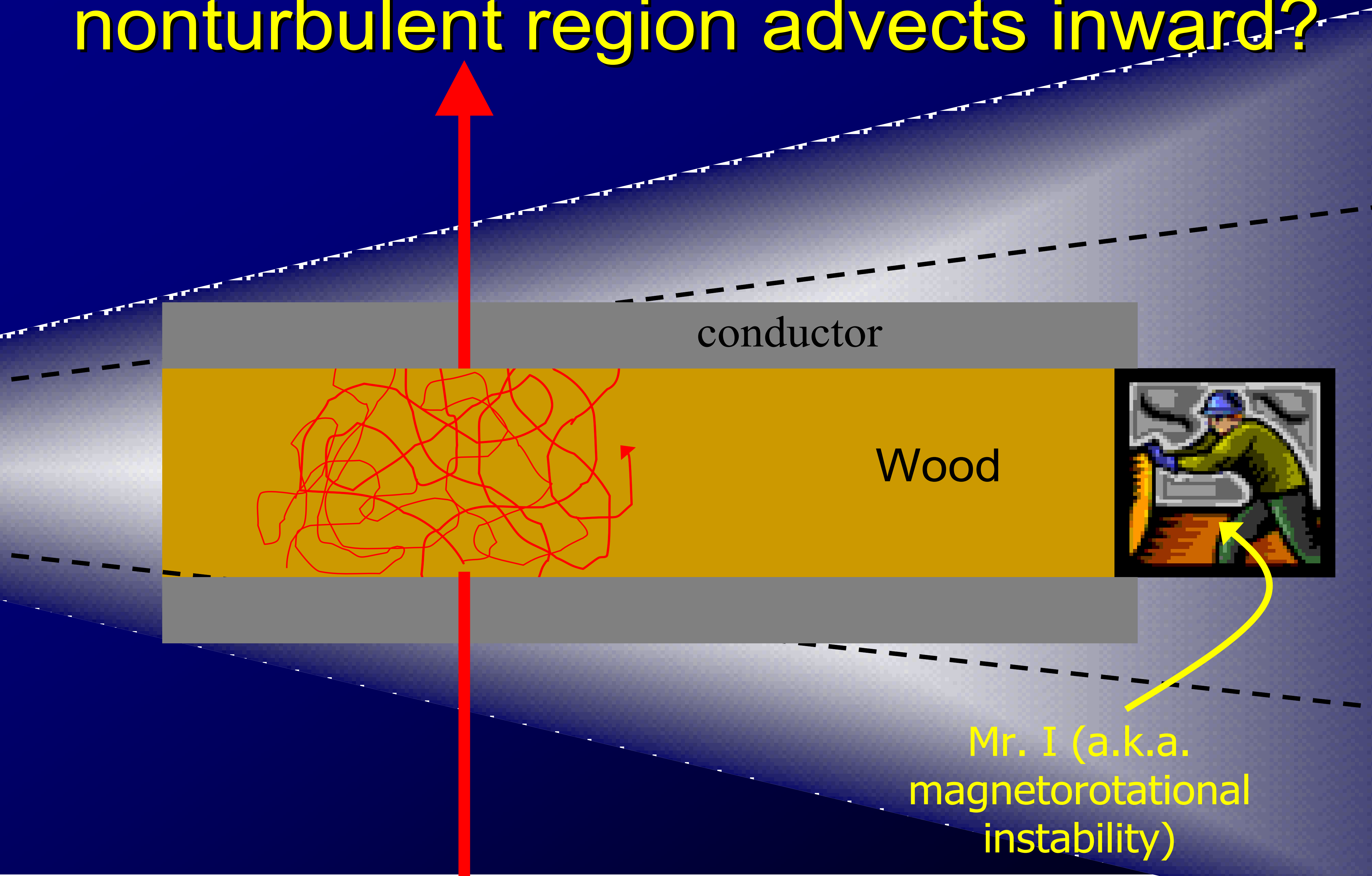
The Basic Idea

Advection in a nonturbulent surface layer



The surface layer can contain enough current to support the large-scale field, even though it only contains a tiny fraction of the disk’s mass

How do we determine if the nonturbulent region advects inward?

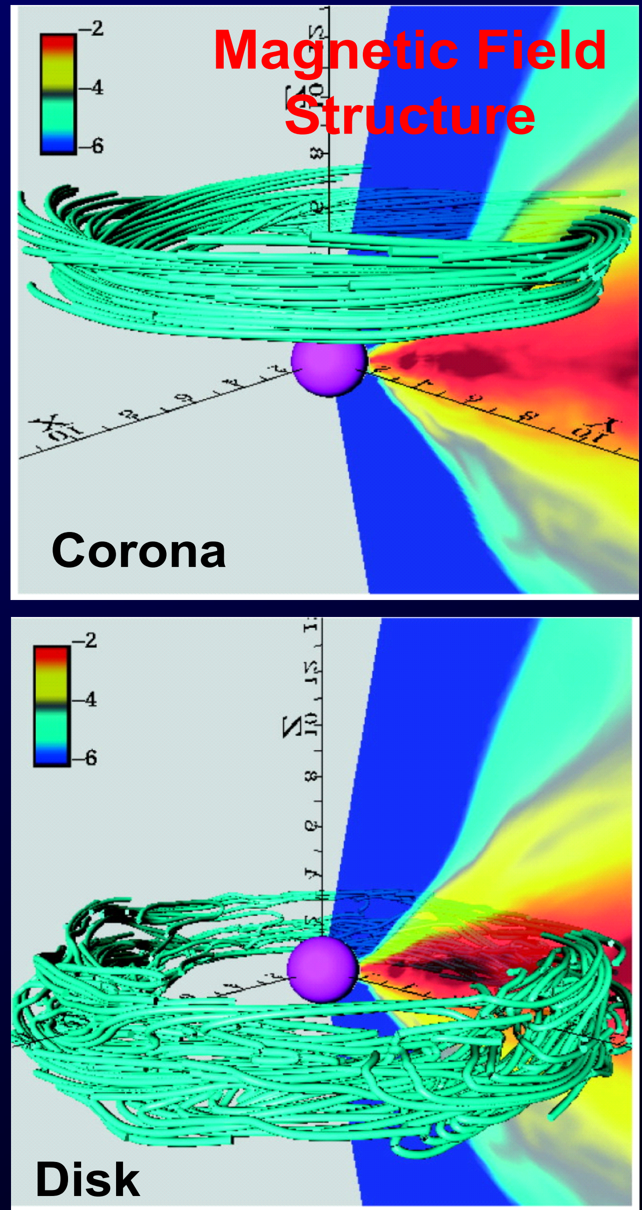


Realistic, Turbulent Accretion Disks

(with the magnetorotational instability)

- **Key point:** MRI is a weak field instability (turbulence shuts off at a height where the field becomes strong compared to the gas)
- Therefore, the field at the base of the nonturbulent region is always strong enough to affect the gas dynamics
- The field is never "too weak" to advect inward (*)

Hirose et al. 2004



Simple geometric condition for field advection

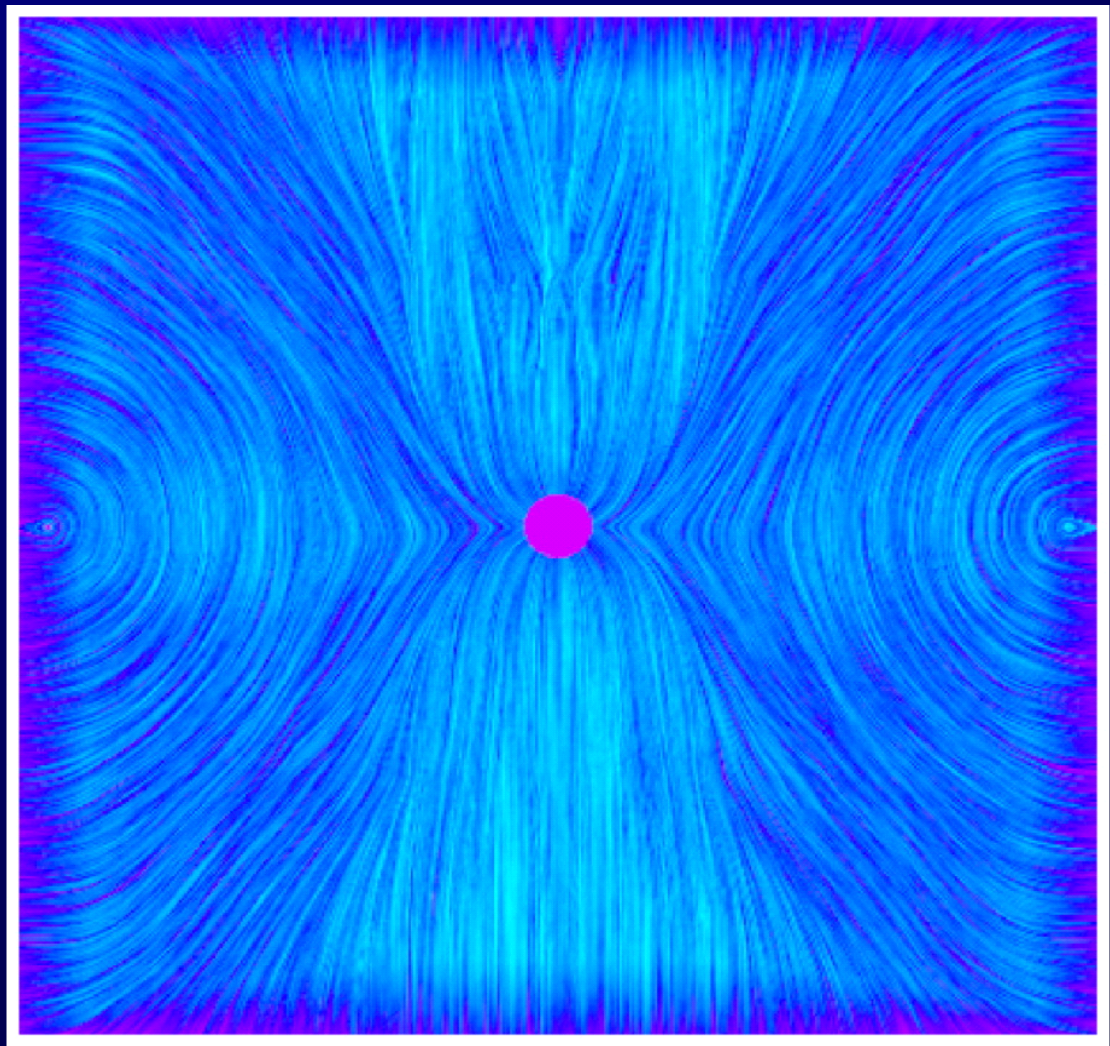
(based on physical effects discussed above)

(vertical magnetic stress / vertical magnetic energy density)
greater than
(vertical velocity / orbital velocity)

- Vertically-stratified MRI shearing box simulations of Miller & Stone (2000) suggest that this will be easily satisfied
- A seed field penetrating the disk ("dipole-type" field) has vertical stress of the correct sign

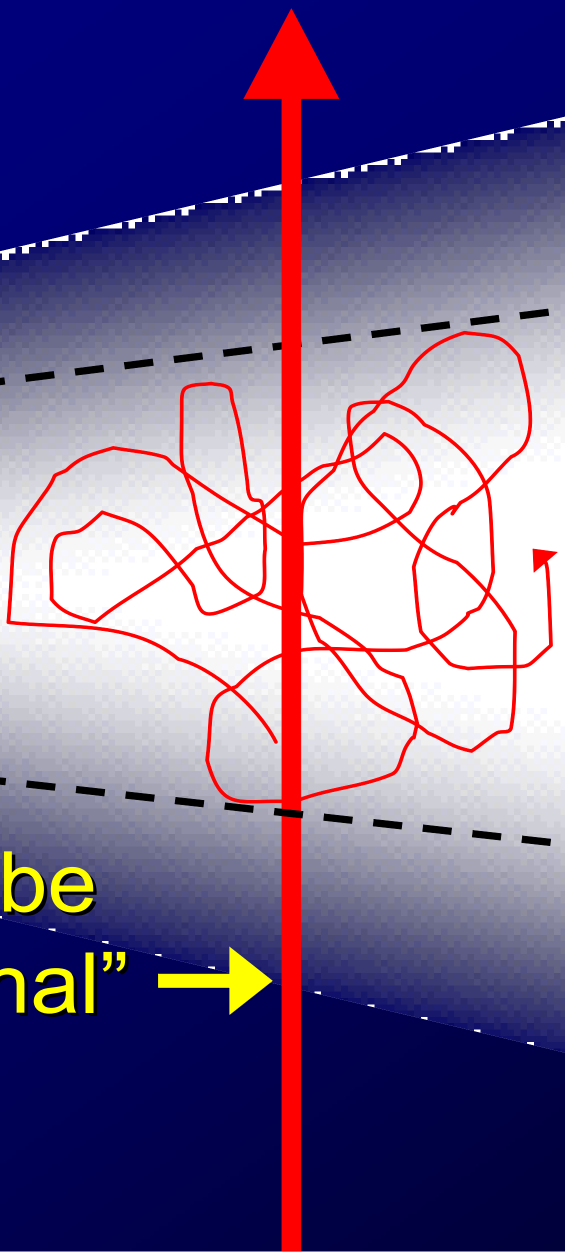
Good things that come from this process

(winds and jets over a wide range of radii in many different compact objects)



Igumenshchev, Narayan & Abramowicz 2003

More good things that come from this process



An "external" field penetrating the disk (~few percent of equipartition) may be needed to drive strong enough turbulence to match observations

(e.g., Pessah et al. 2007, King et al. 2007, Fromang & Papaloizou 2007)

This can be the "external" field! →

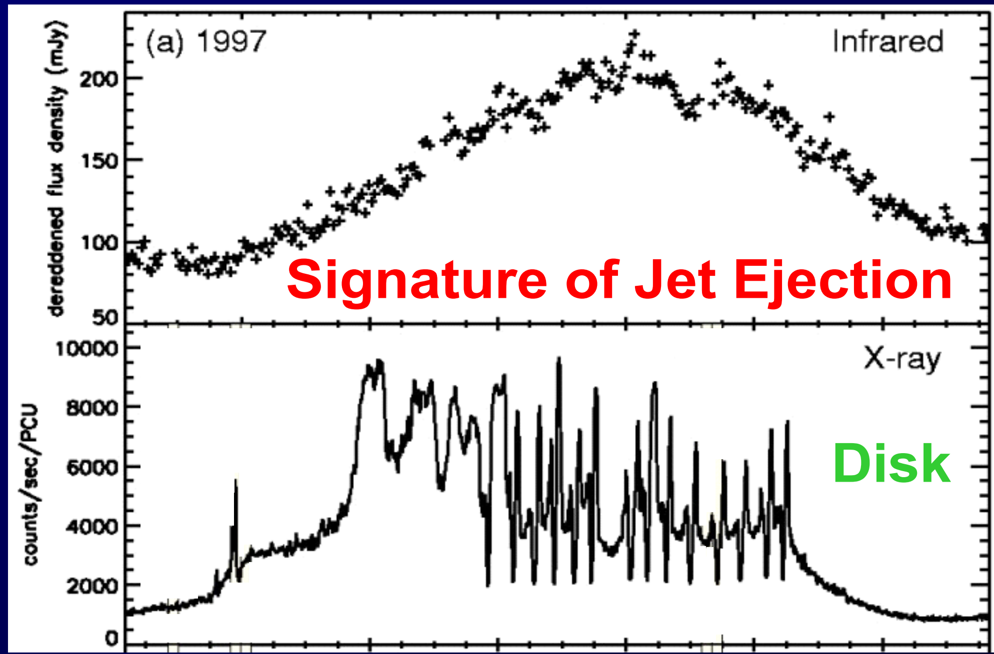
Disk-jet connection

advected magnetic field

jet

turbulent disk timescales

(see also Tagger et al. 2004)



~ 20 min

Observations of GRS 1915+105

(Eikenberry et al. 1998; Rothstein et al. 2005)

Advection/Diffusion of Large Scale Magnetic Field

R. Lovelace, D.M. Rothstein, & A. Koldoba

The main equations are

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathbf{g} + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \mathbf{F}_\nu \ , \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}) \ . \tag{2}$$

$$\nu = \eta = \alpha \ \frac{c_{s0}^2}{\Omega_K} \ g(z) \ , \tag{3}$$

$$\varepsilon = \frac{h}{r} = \frac{c_{s0}}{v_K} \ll 1 \ . \tag{4}$$

The radial component of equation (1) gives

$$\rho \left(\frac{GM}{r^2} - \frac{v_\phi^2}{r} \right) = -\frac{\partial p}{\partial r} + \frac{1}{4\pi} B_z \frac{\partial B_r}{\partial z} + F_r^\nu \ . \tag{5}$$

We define $u_\phi \equiv v_\phi(r,z)/v_K(r)$ and the accretion speed $u_r \equiv$

$- v_r/(\alpha c_{s0})$. This equation then gives

$$\frac{\partial b_r}{\partial \zeta} = \frac{\beta_0}{\varepsilon}\ \bar{\rho}\ (1 - k_p\ \varepsilon^2 - u_\phi^2) + \alpha^2 \beta_0 \frac{\partial}{\partial \zeta}\left(\bar{\rho}\frac{\partial u_r}{\partial \zeta}\right)\ , \qquad (6)$$

where $\bar{\rho} \equiv \rho(r,z)/\rho_0$ with $\rho_0 \equiv \rho(r, z = 0)$. The midplane plasma beta is

$$\beta_0 \equiv \frac{4\pi\rho_0 c_{s0}^2}{B_0^2}\ , \qquad (7)$$

$k_p \equiv -(c_s/c_{s0})^2 \partial \ln p/\partial \ln r$, $p = \rho c_s^2$, and $\zeta \equiv z/h$. Note that $\beta_0 = c_{s0}^2/v_{A0}^2$, where $v_{A0} = B_0/(4\pi\rho_0)^{1/2}$ is the midplane Alfvén velocity. The rough condition for the MRI instability and the associated turbulence in the disk is $\beta_0 > 1$ (Balbus & Hawley 1998). In the following we assume $\beta_0 > 1$.

Similarly, the $\phi-$component of equation (1) gives

$$\frac{\partial b_\phi}{\partial \zeta} = \frac{\alpha\beta_0}{2}\,\bar{\rho}\,\left(3\varepsilon k_\nu g - u_r\right) - \frac{\alpha\beta_0}{\varepsilon}\frac{\partial}{\partial\zeta}\left(\bar{\rho}\frac{\partial u_\phi}{\partial \zeta}\right)\,, \qquad (8)$$

where $k_\nu \equiv \partial \ln(\rho c_{s0}^2 r^2)/\partial \ln(r)$ is of order unity. The dominant viscous force contribution in this equation is $F_\phi^\nu = -\partial T_{\phi z}^\nu/\partial z$ with $T_{\phi z}^\nu = -\rho\nu\partial v_\phi/\partial z$ and this gives the second derivative term.

The $z-$component of equation (1) gives

$$\frac{\partial p}{\partial \zeta} = -\rho \zeta c_{s0}^2 - \frac{\rho_0 c_{s0}^2}{2\beta_0} \frac{\partial}{\partial \zeta} \left(b_r^2 + b_\phi^2 \right) \ . \tag{9}$$

The term involving the magnetic field describes the magnetic compression of the disk because $b_r^2 + b_\phi^2$ at the surface of the disk is larger than its midplane value which is zero (Wang et al. 1990). For $\beta_0 \gg 1$ the compression effect is small and can be neglected.

For specificity we consider the adiabatic dependence $p \propto \rho^\gamma$ in the vertical direction with $\gamma = 1 - 5/3$. This can be written as $p = \rho c_{s0}^2 (\rho/\rho_0)^{\gamma-1}$. This gives

$$\tilde{\rho} = \frac{\rho}{\rho_0} = \left(1 - \frac{(\gamma-1)\zeta^2}{2\gamma}\right)^{1/(\gamma-1)} , \tag{10}$$

for $\beta_0 \gg 1$. The density goes to zero at $\zeta_m = [2\gamma/(\gamma-1)]^{1/2}$. However, before this distance is reached the MRI turbulence is suppressed, and $g(\zeta)$ in equation (3) becomes very small.

The toroidal component of equation (2) gives

$$\frac{\partial b_r}{\partial \zeta} = \frac{u_r}{g} . \tag{11}$$

Because $g(\zeta)$ tends to $\epsilon \ll 1$ as ζ increases (from a value say ζ_c of order unity), u_r must also tend zero beyond this distance in order for $\partial b_r/\partial\zeta$ to remain finite.

The other component of equation (2) gives

$$\frac{\partial u_\phi}{\partial \zeta} = \frac{3}{2}\varepsilon b_r - \alpha\varepsilon \frac{\partial}{\partial \zeta}\left(g\frac{\partial b_\phi}{\partial \zeta}\right) . \tag{12}$$

Combining equations (6) and (11) gives

$$u_r = \frac{\beta_0}{\varepsilon}\tilde{\rho}g\left(1 - k_p\varepsilon^2 - u_\phi^2\right) + \alpha^2\beta_0\frac{\partial}{\partial \zeta}\left(\tilde{\rho}\frac{\partial u_r}{\partial \zeta}\right) . \tag{13}$$

$$\alpha^4\beta_0^2\frac{\partial^2}{\partial\zeta^2}\left(g\frac{\partial}{\partial\zeta}\left(\bar{\rho}\frac{\partial}{\partial\zeta}\left(\frac{1}{\bar{\rho}}\frac{\partial}{\partial\zeta}\left(\bar{\rho}\frac{\partial u}{\partial\zeta}\right)\right)\right)\right)$$
$$-\alpha^2\beta_0\frac{\partial^2}{\partial\zeta^2}\left(g\frac{\partial}{\partial\zeta}\left(\bar{\rho}\frac{\partial}{\partial\zeta}\left(\frac{u}{\bar{\rho}g}\right)\right)\right)-\alpha^2\beta_0\frac{\partial^2}{\partial\zeta^2}\left(\frac{1}{\bar{\rho}}\frac{\partial}{\partial\zeta}\left(\bar{\rho}\frac{\partial u}{\partial\zeta}\right)\right)$$
$$+\alpha^2\beta_0^2\frac{\partial^2}{\partial\zeta^2}\left(\bar{\rho}g(u-3k_\nu\varepsilon g)\right)+\frac{\partial^2}{\partial\zeta^2}\left(\frac{u}{\bar{\rho}g}\right)+3\beta_0\frac{u}{g}=0\ . \quad (15)$$

This is our main equation. The equation can be integrated from $\zeta = 0$ out to the surface of the disk where $u = 0$. Because of the even symmetry in ζ, the odd derivatives of u are zero at $\zeta = 0$, but one needs to specify $u(0)$, $u''(0)$, and $u^{iv}(0)$. Clearly, a "shooting method" can be applied where the values of $u(0)$, $u''(0)$, and $u^{iv}(0)$ are adjusted so as to give $u = 0$ on the disk's surface. Once $u(\zeta)$ is calculated, equations (6) and (11) can then be used to obtain $b_\phi(\zeta)$ and $b_r(\zeta)$.

We restrict our attention to physical solutions which have net mass accretion,

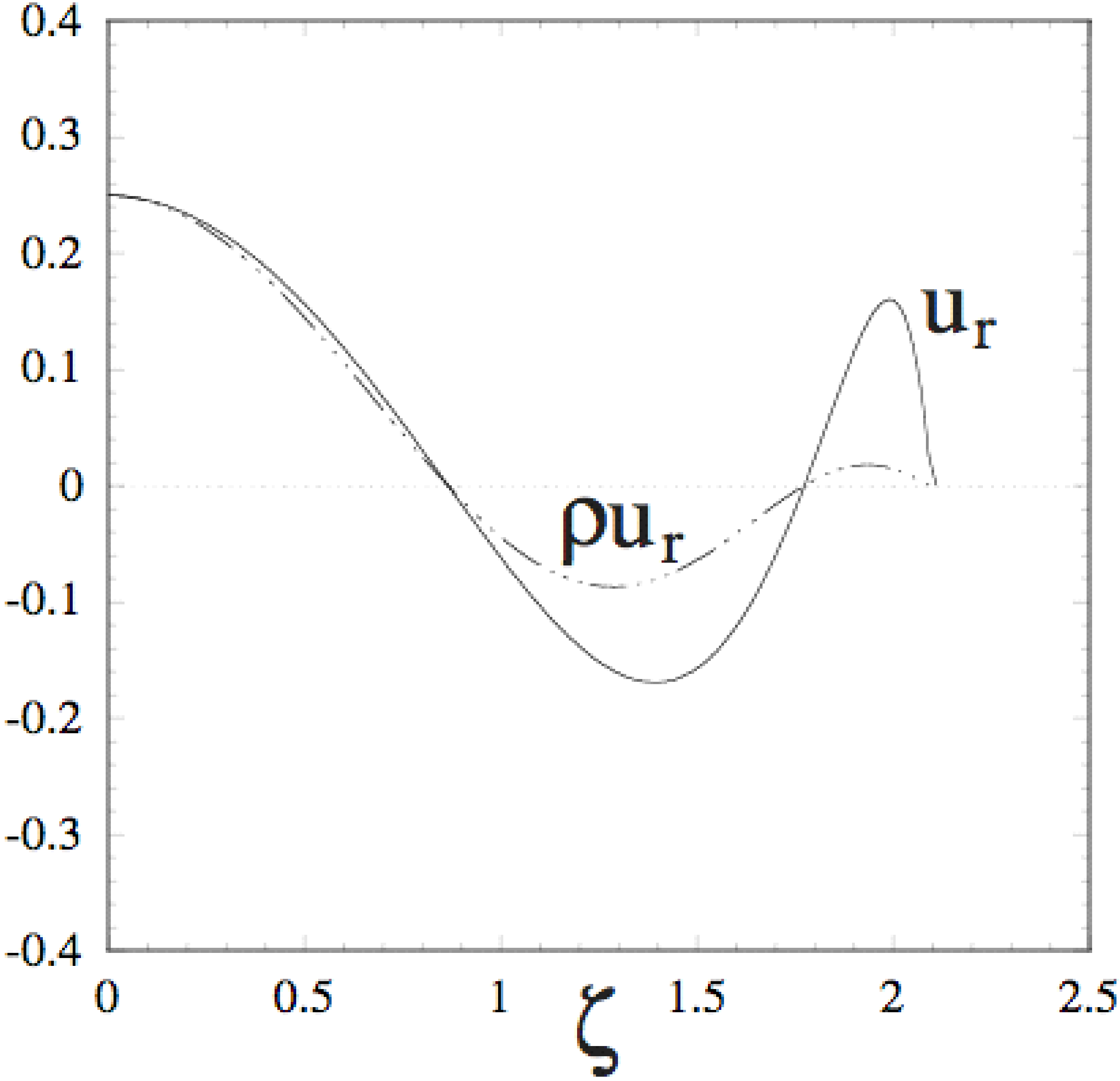
$$\dot{M} = 4\pi r h \rho_0 \alpha c_{s0} \int_0^{\zeta_m} d\zeta \ \tilde{\rho} u > 0 \ ,$$

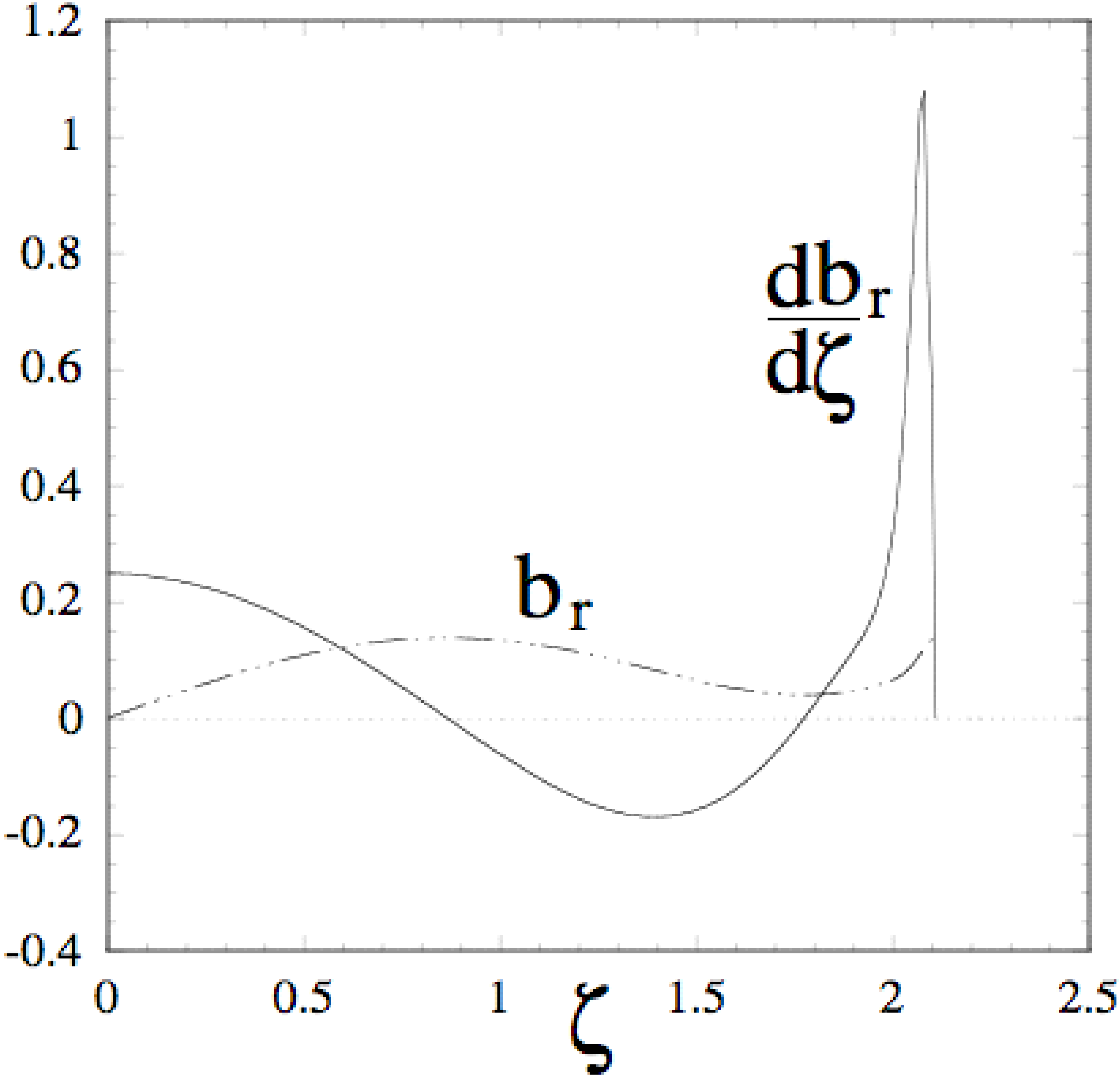
and have $b_\phi < 0$ on the disk's surface. This condition on b_ϕ corresponds to an outflux of angular momentum and energy rather than the reverse.

Figures 1 and 2 show results for an illustrative solution of equation (15). We have taken $\beta_0 = 100$, $\alpha = 0.1$, $\varepsilon = 0.05$,

$\gamma = 5/3$, $\zeta_m = 2.24$, $k_p = 1$, and $k_\nu = 1$. The parameters of g are $\zeta_c = 2$ and $\Delta z/h = 0.05$. For this solution we have chosen $u(0) = 0.25$ and adjusted $u''(0)$ and $u^{iv}(0)$ in a shooting method to give $u = 0$ on the disk's surface. The values found are $u''(0) = -0.75$ and $u^{iv}(0) = 1.2$. The density-weighted accretion speed is $\overline{u} = 0.0661$.

In general we can use a shooting method to adjust the values of $u(0)$, $u''(0)$ and $u^{iv}(0)$ to give $u = 0$ on the disk's surface as well as specified values of b_r and b_ϕ on this surface.


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.5
1
1.5
2
2.5
ζ
u_r
ρu_r



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
ζ
$\frac{db_r}{d\zeta}$
b_r

Balbus, S.A., & Hawley, J.F. 1998, Rev. Mod. Phys., 70,1

Bisnovatyi-Kogan, G.S., & Lovelace, R.V.E. 2007, ApJ, 667, L167

Blandford, R.D., & Payne, D.G. 1982, MNRAS, 199, 883

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Li, Z.-Y., 1995, ApJ, 444, 848

Lovelace, R.V.E., Romanova, M.M., & Newman, W.I. 1994, ApJ, 437, 136

Ogilvie, G.I., & Livio, M. 2001, ApJ, 553, 158

Shakura, N.I., & Sunyaev, R.A. 1973, A&A, 24, 337

Conclusions

- We predict that a vertical field threading an MRI-unstable disk will advect inward under general conditions that we derive.
- This process may help explain the origin of disk turbulence.
- It also provides a simple mechanism to generate strong magnetic fields needed for jets and outflows.